UNIVERSITY OF ALBERTA

VELOCITY DISTRIBUTION OVER DUNE BEDFORMS

BY

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ABSTRACT

Non-uniform flow characteristics over fixed dune bed is studied, using previously measured velocity profiles and bed shear stresses. The velocity profiles are measured using the Laser Doppler Anemometer . The behavior of the flow are analysed in two ways. The difference of velocity and bed shear distribution from uniform flow described by the classical log law velocity distribution is investigated. The velocity is non-dimensionalized using mean velocity, surface velocity, and peak perturbation velocity. The length scale is non-dimensionalized using dune-length. The non-dimensional forms of the velocity profile are plotted versus the relative depth of flow. A relation between velocity scale and the bed shear stress is found. Second, the distribution of velocity in the outer region is studied by analogy to the plane wall-wake theory. Using the velocity distribution, the size of obstacle and other geometric properties of the channel, the wall wake analysis gives mean velocity profiles, wall shear stress, decay of velocity defect, and growth of the half-width of the wake in convenient forms. These parameters along with a set of shape functions describe the velocity profile.

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List of Symbols

Symbol	Description
b	Length scale;
$\bar{b}$	Half width of wall wake;
C_D	drag coefficient;
F	Froude number;
g	Acceleration due to gravity;
H	Mean flow depth;
h	Height of dune;
k_s	Bed roughness due to sand grains;
ℓ	Prandtl mixing length;
L	Wave length of the bed form;
L_r	Reattachment length;
p	Pressure;
Q	Discharge;
R	Reynolds number ;
S	Mean slope;
U	Depth-averaged velocity ;
U_o	Freestream Velocity;
u	Local point mean velocity;
u_1	Velocity defect;
u_{1m}	Maximum velocity defect;
u_c	Calculated mean velocity;
$-\overline{u'v'}$	Reynolds' shear stress;
u_*	Shear velocity;

U_{*c}	Total shear velocity;
$(u_c - u)_s$	Differences in surface velocity;
$(u_c - u)_p$	Differences in peak perturbation;
x	Streamwise coordinate measured from bed-form crest;
y	Elevation above mean bed level;
z	Elevation from arbitrary datum;
β	Angle of local dune slope;
η	$\frac{y}{b}$
τ	Shear stress;
τ_o	Wall shear stress;
τ_c	Calculated wall shear stress;
κ	Von Karman constant (≈ 0.4);
ν	Kinematic viscosity;

CHAPTER 1. INTRODUCTION

The purpose of the study reported herein was to investigate the detailed characteristics of non-uniform flow over dune and ripple shaped bed forms. The detailed characteristics of non-uniform flow is important in calculating resistance and erosion of alluvial channels. Prediction of bed form evolution, sediment transport rate and actual channel stage are the ultimate applications of this knowledge. Existing techniques apply bed stress relations based on depth averaged velocity and uniform flow assumptions.

As in boundary layers with pressure gradients, larger bed stresses for accelerating flows and reduced stresses, (possibly negative in a separated region) for decelerating flows are expected. The main results desired from this study are: identification of a disturbance velocity scale, evaluation of the behavior of this scale with distance and the evaluation of relationship between the velocity scale and bed shear stress.

This study uses the Laser Doppler Anemometer measurements performed at Delft (1988) and by Lyn (1993). The behavior of the flow over the dune was evaluated non-dimensionally from these empirical results in two distinct analyses:

1- Investigation of difference of velocity and shear distribution from uniform flow described by the classical log-law velocity distribution.

2- Determination of the distribution of velocity in the outer region by the plane wall-wake theory and the log-law for the inner region.

In chapter 2, a brief review on velocity distribution and bed shear stress is presented. Open channel flows over dunes are discussed in addition to the plane wall-wake. A summary of the experiments used in the present study is given. The difference of the

velocity distribution for disturbed flow over dune from uniform flow is discussed in chapter 3 . A similarity analysis based on an analogy with the plane wall-wake is presented in chapter 4.

CHAPTER 2. LITERATURE REVIEW

2.1 Introduction

A survey of the existing literature on turbulent flow involving velocity distribution and bed shear stress reveals that such investigations are rather limited to the case of uniform open-channel flows. In this chapter, a brief review on the earlier works shows the level of our knowledge on the dynamics of flow in open channel flow. The review is restricted to velocity distribution and bed shear stress.

2.2 Earlier Work On Shear Stress and Velocity

The flow about a solid body is characterized by boundary layer theory generated by dividing it into two regions:

a) wall (inner) region in the neighborhood of the body in which viscocity is very important.

b) core (outer) region where the viscocity is ignored.

Prandtl (1914) showed from his experiments that flow in a boundary layer can be laminar or turbulent. The transition affects separation and drag. The upper layer with a constant velocity V_m is considered as the core (outer) layer. In the bottom layer which called boundary layer; the velocity of the fluid increases from zero at the wall and finally approaches the value corresponding to the frictionless flow (free-stream). The boundary layer is based on Prandtl's theory.

The theoretical formula developed by Prandtl and Karman and verified by the measurements of Nikuradse have resulted in rational formulas for velocity distribution and bed shear stress for turbulent flow in circular pipes. G. H. Keulegan (1938) applied the pipe formule to study open channel flow, with certain assumptions regarding the

effects of secondary currents and free surface. Also, the hydraulic radius was chosen as the characteristic length. Similar rational formulas for open channel were then developed. Keulegan verified this formula using the experimental work of Bazin.

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln(\frac{yu_*}{\nu}) + A \qquad (2.1)$$

Where u is the local point mean velocity, u_* is the shear velocity, κ is the Von Karman constant, y is the distance above mean bed level, and ν is the kinematic viscosity.

Recent studies on the log-law has led to the conclusion that it gives good results in the wall region but there is a deviation in the upper layer. Adjusting κ and A in Eqn. (2.1) is not sufficient to remove this deviation. However, a new function referred to as a wake function can be added to the log-law as shown in equation (2.2). Many wake functions have been proposed to date (Coles, 1956, I.Nezu and H. Nakagawa, 1993) and appear in the velocity distribution formula as:

$$\frac{u}{u_*} = \frac{1}{k}\ln(\frac{yu_*}{\nu}) + A + w(y/H) \qquad (2.2)$$

Where H is the mean flow depth and w is the wake function.

Consequently, the velocity close to bed can be calculated by the log-law, and in the outer region $(y/H > 0.2)$, the log-wake law is applicable. In the present study the values for k and A (irrespective of flow type) in the above equations are Nikuradse's values for smooth-pipe flow. Good agreement with the linear from of the log-law was showed experimentally in the wall region by Steffler, Rajaratnam and Peterson (1985). Near the water surface, the velocity profiles were found to drop off from the log-law line (dip).

2.3 Effects Of Free Surface

The effect of a free surface on turbulence is appreciable in the calculation of open channel flows. The vertical movement of eddies is affected by the free surface. Nezu (1977) analyzed the flow approaching the critical condition from the tranquil state ($Fr \rightarrow 1$), and found that the effect of free surface on turbulence is very small but important for near critical flows. The effects of surface tension on preventing surface waves can be considered effects of flows with small Froude number by the free surface.

The importance of the free surface can thus be considered when the vertical velocity $v(t)$ are measured. They are damped by the free surface. These effects could be supported by measurements by Smutek(1969), Nakagawa et al. (1975) and Komor et al. (1982), and also phenomenologically by Hunt (1984). The damping of the vertical fluctuations causes v'/u_* to decrease rapidly towards the free surface.

Experimental investigations by Nezu and Rodi (1986) indicate satisfactory agreement with ν_t (eddy viscosity) approaching zero at the free surface in open channel flow. Whereas it is not zero at the symmetrical axis of closed channel flow. This shows a discrepancy between open-channel and closed-channel flow.

2.4 Open Channel Flows Over Dunes

The American Society of Civil Engineering (ASCE, 1969) Task Committee on Bed Forms defined ripples and dunes as:

a) Ripples are small bed forms with wave-length less than approximately 0.33 m (1 ft) and height less than 0.03 m (0.1 ft). When the flow velocity increases slightly higher than that required for initiation of sediment motion but lower than that for flat bed or antidunes, ripples occur.

b) Dunes are bed forms smaller than bars but larger than ripples. When the flow velocity and transport rate increase higher than those for ripples but smaller than those for antidunes, dunes occur.

Anderson (1953) studied the wave lengths of bed forms in terms of Froude number by use of a potential flow analysis of flow over a wavy bed.

Kennedy (1963) developed an analytic model of free-surface flow over an erodible bed. He, as in the case of Anderson (1953), employed potential flow theory and the lag distance concept. Kennedy concluded that the Froude number, the depth of the flow, and the lag distance are the major factors that determine the type of bed form and its wave-length. Besides these factors, sediment transport rate affects the velocity of the bed features.

Raudkivi (1963) with laboratory experiments showed that the mean velocity over the crest is 1.09 times of the mean velocity over the flat bed. Also, he showed that by using his own data the main stream shear velocity appears to be equal to or less than that over the flat bed. By comparing the maximum surface drag over the crest with that on the flat bed, it was concluded that they are almost of the same magnitude. He indicated that this drag is probably a big influence on the rate of transport. His measurements show an increasing surface drag in the downstream direction from trough to crest. He concluded that both the applied surface drag and the turbulent agitation on the crest of ripples affect the rate of entrainment and transport. The maximum value of the surface drag is outside the direct influence of the interface turbulence. A region of low pressure is seen over the ripple crest region rather than in the lee of the ripple only.

Engelund (1965) investigated the flow over a fixed and movable sinusoidal bed. His analysis showed how dunes (or antidunes) are formed. The bed configuration diagram, the Froude number versus $V / U_f^{'}$ in which $U_f^{'}$ is the effective friction velocity, were plotted in his report.

Hayashi (1970) theoretically studied the formation of dunes and antidunes on the erodible bed of an open channel. He presented a physical model for the transport of sediment over a wavy bed and concluded that the lag distance introduced by Kennedy (1963) plays a central role in the formation of dunes and antidunes.

Based on laboratory experiments, Yalin and Karahan (1979) presented steepness of sedimentary dunes in graphical forms. They also concluded that the flow affects the sand waves simultaneously in two conflicting ways:

> "On the one hand it appears to "build them up," perhaps by eroding their downstream side and thus by rendering the increment of the "scour depth" Δ ($\approx \delta$) (constructive action); on the other, it tends to "wash them out" perhaps by exerting the drag force action on their upstream surfaces (destructive action). In the early stages, the former action appears to be dominant, in the later stages the latter."

Fredsoe (1982) has developed a mathematical model to calculate the shape and dimensions (length, height) of sand dunes in rivers. His studies are based on the measured bed shear stress distribution downstream of a rearward-facing step. This model can be used to calculate the flow-resistance curves in alluvial streams.

Based on 1500 reliable flume and field data, Leo C. van Rijn (1984) presented a method which determines the dimensions of bed-forms as well as their equivalent roughness. The Chezy-coefficient in the dune and plane bed regimes can be calculated by this model.

Several experimental studies of flow over dune bed forms have been done. The experimental works by Einstein (1937), Tani (1957), Walker (1961), Raudkivi (1963), Vanoni and Hwang (1967), Kennedy (1969, 1980), Chang (1970), Crickmore (1970), Riafi and Smith (1971), Onys (1973,1974), Fredsoe (1975) Vittal et al. (1976), Mclean (1976), Yalin (1977, 1985), Yalin and Karahan (1979), Etheridge and Kemp (1979), Engelund and Lau (1980), Patankar (1980), Felhman (1985), and Mahmood et al. (1987) are some of the many studies that can be found in the sediment literature.

Various analytical and numerical solutions have also being published. Haque (1970), Mercer (1971), Rodi (1972), Lee et al. (1974), Launder et al. (1975), Puls et al. (1977), Richards and Taylor (1981), Haque and Mahmood (1983, 1986), Mendoza and Shen (1985a, 1985b), and Mclean and Smith (1986) are some of the works that can be found in the literature.

The only detailed measurements available to the writer are measurements of the characteristics of turbulent flows over dunes were performed at Delft (1988) and by Lyn (1993).

One of the objectives of experiments of the Delft study was to obtain a data base for the verification of mathematical computer programs which predict the alluvial bed roughness in rivers under flood conditions. In this study, instantaneous flow velocities in the longitudinal and vertical directions and the instantaneous bed pressures were measured.

Lyn (1993) obtained turbulence measurements in open channel flows over artificial bed form in order to evaluate the similarity structure of the outer region of dune flows and to determine the appropriate scaling of the turbulence characteristics with other simple flows, e.g., the flat-bed flow or the backward-facing step.

The experiments by Lyn were conducted in flows of constant depths over two types of bed form . However, the experiments at Delft were conducted using two depths of flow and a single type of bed form within a restricted range of Froude number, $F = U / \sqrt{gH}$, where U is the depth averaged velocity, H is the mean flow depth.

A summary of the more important experimental conditions is given. For the purpose of presentation, the results are presented under two headings: (1) Lyn's Data and (2) Delft Data. Subsequent sections describe briefly the experimental apparatus, techniques, and instrumentation used in these studies.

2.4.1 Lyn's Data

The data presented by Lyn (1993) was obtained from experiments in an adjustable-slope recirculating flume approximately 13 m long and 0.27 m wide. The main dimensions and features of this flume are given in table 3.1. The same flume was used for all experiments. The velocity measurements were obtained using a two-component laser-doppler velocimeter. (LDV)

The first type of bed form was ripples. The height of a particular ripple was taken as the difference in elevation between the ripple crest and elevation of the bottom of the flume. The bed consists of a periodic array of 45° triangular elements of amplitude $h = 1.2 \pm 0.1$ cm and wavelength $L = 15 \pm 0.3$cm (to be referred to as Type I). Lyn assumed this ripple to be a dune, while realizing that this is not a realistic as a dune model. However, it helps to understand the effects of bed geometry on turbulence characteristics. The second type of bed form used in the experiments by Lyn (TypeII) was an idealized dune. The dune was not like a real dune, because the top of the dune did not exhibit a flat plateau. Although ripples and dunes both had a 45° angle for the downstream face and the same wavelength, Lyn distinguished them using upstream geometry.

The flume was made from treated wood, and a layer of 0.25 mm sand was glued to the bottom of the flume.

The width-to-depth ratio of the flows was constant at 4.4 and a constant depth $H = 6.1 \pm 0.2$ cm was used. A fully developed channel flow was established by choosing the ratio of flow development lengths to depth and to length as approximately 150 and 60, respectively.

The dune (ripple) steepness, h/L, was 0.08 in the experiments by Lyn. This ratio has an intermediate value between those indicated by Vanoni and Hawang (1967) and Fredsoe (1982). Vanoni and Hawang (1967) showed that h/L may often exceed 0.1 and by using experiments from Yalin (1972), Fredsoe (1982) proposed a ratio of 0.06. Two experiments were performed using Type II bed forms. One of them had the same Froude number as Type I and the second one was performed using the same shear Reynolds number as Type I.

2.4.2 Delft Data

Experiments were conducted in the sand flume. Before the experiments were commenced, the flume was equipped as follows: (independent variables)

-constant width of 1.50 m

-constant discharge per experiment

-constant energy

-constant slope per experiment, and

-constant water temperature of about $18^{\circ}C$ per experiment

A layer of sand grains , 1 to 2 times D_{50}, viz. 1.6 to 3.2 mm, was glued on the concrete dunes (using Tooski glue).

Measurements of $\overline{U}, \overline{W}, \overline{u'u'}, \overline{w'w'}, \overline{u'w'}$ above sand plastered concrete dunes (L=1.60 m, H=0.08 m) at a distance 0.37 m from the wall (because of the optical setup and the width of the flume) were recorted. Flow intensities were measured at 16 cross-sections along a dune. Also, the velocity and bed pressure measurements were obtained. From these experiments, the following parameters were determined: shear stress velocity, location of reattachment point, local energy-level slopes, turbulent kinetic energies, eddy viscosities, and turbulent mixing lengths,

Because of the bed material which was chosen, the bed roughness and the shear stress velocity (U_*)could be calculated. A value of 0.4 was used for the Von Karman (κ) and in experiments T_5 and T_6, the k_s value was chosen as 2.5 mm which is the grain size and these used to determine the shear velocity.

It was assumed that the boundary layer along the dune surface in experiments T_5 and T_6 is a turbulent boundary layer corresponding to uniform channel flow. On the basis of these assumptions, the formulas for the boundary layer theory was applied to obtain values for τ_b and $\overline{U}$.

2.5 Wall-wake

Schlichting (1930) was the first who presented the plane wake theory based on Prandtl's mixing length hypothesis. Since then many studies have been done. However, very few works were in the field of wall-wake. Sforza and Mons (1969) investigated the

effects of large inviscid disturbance caused by leading edge obstacle. In this study, the flow behind an obstacle was classified into 3 regions namely; the recirculation region, the characteristic decay region, the asymptotic decay region.

Rajaratnam and Rai (1979) used a two-layer model to describe the far-wake part of plane turbulent wall wakes. The plane wake equation was used to describe the velocity distribution in the outer region and the log-law equation in the inner region.

However, no study has been done on flow over dunes using a wall-wake theory (known to the writer).

2.6 Conclusions

The aims of this report are to present some experimental results concerning the velocity distribution and bed shear stress in open channel flows and to detect the influence of non-uniform flow. Two experiments which were mentioned above in relation to open channel flows are used as a basis for developing our understanding of the velocity and the bed shear stress.

Two series of experiments discussed were qualitative and quantitative, and were undertaken with specific intentions. As can be seen, Lyn's data aimed at identifying the appropriate scaling of the turbulence characteristics with the other simple flows. However, the main aim of Delft measurement was to verify the mathematical computer programs which predict the alluvial bed roughness in rivers. In the next chapter, this report looks at these studies from a different perspective.

Parameter (1)	Lyn's study			Delft Study	
	Case 1 (2)	Case 2 (3)	Case 3 (4)	T5 (5)	T6 (6)
Bed form	Type I	Type II	Type II	--	--
Flow depth H (cm)	6.1	6.1	6.1	25.2	33.4
Energy Slope, $s(10^{-3})$	5.25	1.45	4.0	.96	.95
Mean Velocity, U(cm/s)	25.4	26.9	55.1	44	55
h/L	0.08	0.08	0.08	0.052	0.052
h/H	0.2	0.2	0.2	0.33	0.25
$F = U / \sqrt{gH}$	0.33	0.35	0.71	0.26	0.29
$R = UH / \nu(10^3)$	15.5	16.4	33.8	100.8	177

Table 2.1. Experimental parameters (Source: Lyn (1993))

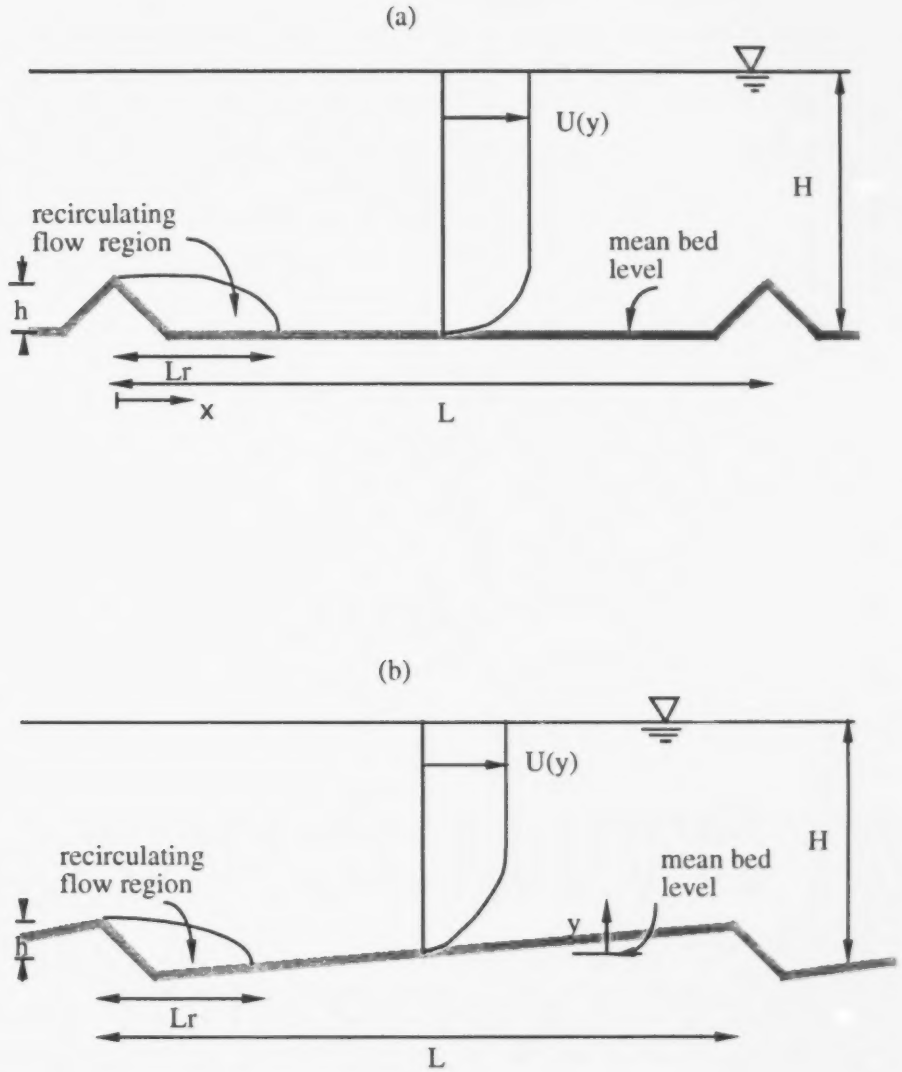


Figure 2.1 Defination sketch of flow and bed forms studied:
(a) Type I bed form; (b) Type II bed form
(source : Lyn [1993])

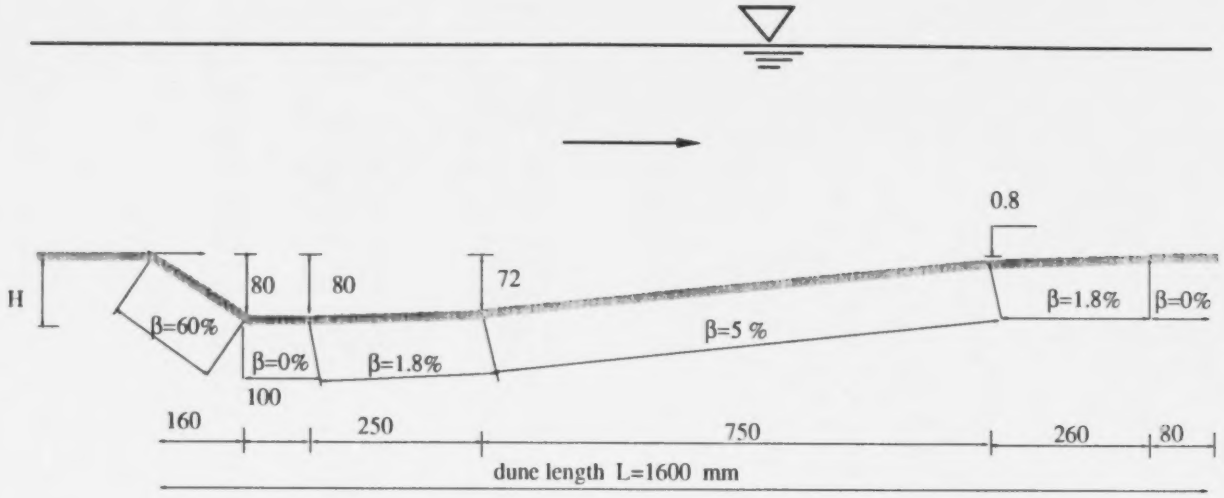


Figure 2.2. Concret dune profile
(source: Delft [1988])

CHAPTER 3: VELOCITY PERTURBATION ANALYSIS

3.1 Introduction

One of the important studies in sediment transport is that of understanding the behavior of turbulent flows over dunes, particularly with regard to their mean velocity distribution and shear stress. The goal of the investigation herein described is to investigate the velocity distribution of turbulent flow over dunes. In attempting to achieve this goal a similarity analysis of velocity distribution is presented which helps to illustrate the relationship of the discrepancy between non-uniform and uniform flow to the depth of flow.

It is helpful to distinguish and define the three subregions of the turbulent structure of open channel flows:

(i) Wall Region $(\frac{y}{H} \leq 0.2)$

(ii) Intermediate Region $(0.2 < \frac{y}{H} \leq 0.6)$

(iii) Free-Surface Region $(0.6 < \frac{y}{H} \leq 1.0)$

3.2. Velocity Distribution

As mentioned earlier, we are trying to find a simple relationship in non-uniform flow over dune in terms of macroscale (depth, mean velocity and bed slop).

The quantity the shear velocity u_* is calculated by

$$\frac{U}{u_*} = 2.5 \ln \frac{H}{K} + 6.2 \qquad (3.1)$$

$$K = 3.3 \frac{\nu}{u_*} + K_s \qquad (3.2)$$

in which U=the mean velocity K_s=bed roughness, H=the depth of the flow, ν=the kinematic viscosity, and κ=0.4 the Von Karman constant

For the uniform flow in a smooth turbulent flow, the longitudinal velocity ,u, can be expressed in terms of the log-law equation by

$$\frac{u_c}{u_*} = \frac{1}{\kappa}\ln\left(\frac{yu_*}{\nu}\right) + A \qquad (3.3)$$

Figures 3.1, 3.2 and 3.3 show a comparison of the measured data and the calculated mean velocity by use of log-law equation at different locations along the flow stream. It can be seen that there is a similarity at any section for any run. The maximum defect velocity decreases downflow and the height of the location $\left(\frac{y}{H}\right)$ of the mean velocity increases.

From Figures 3.13, 3.14 and 3.15 some resemblance between different runs can be seen. As the depth of the flow increases, the maximum defect velocity close to the bed increases and the relative elevation $\left(\frac{y}{H}\right)$ of the point where the velocity equals the mean velocity decreases. Figures 3.13, 3.14 and 3.15 show that the mean velocity occurs for Lyn's Data at $\left(\frac{y}{H}\right) = 0.40$ and for T5 at $\left(\frac{y}{H}\right) = 0.30$ and for T6 at $\left(\frac{y}{H}\right) = 0.26$. As a result, the value of the depth of the flow is one factor that affects the maximum defect velocity in turbulent flow and where the mean velocity occurs.

Figures 3.4 to 3.9 show differences from uniform flow case 1, 2, 3 from Lyn and Delft. Figures 3.10 to 3.15 are normalized differences in velocity with surface velocity and Figures 3.16, 3.17 and 3.18 with peak perturbation. All Figures confirm the results.

The peak perturbation versus wave length was plotted in Figure 3.19. From Figure 3.19 it was found that for $\frac{X}{L} \geq 0.2$ the data are well described by a curve and peak perturbation tends to zero.

Three subregions of the turbulent structures of open channel flow has to be considered in this model. As we know, log-law was found from assumptions of pipe flow. From Figures 3.14 and 3.15 it is seen that the mean velocity the same as the vertical turbulence intensity is affected by the free surface. The maximum velocity is shifted to below the free surface, and bursting phenomena and inner variables affect the calculation of the mean velocity close to the bed.

3.3 Bed Shear stress

Lyn (1994) presented a valuable discussion on his own data and the Delft data. He studied the velocity defect and the Reynolds shear stress profile from the flat-bed profiles for region of flow far from the bed.

Figures 3.20 and 3.21 show that flow acceleration near the bed makes the bed shear stress increase very rapidly and it becomes almost a constant downstream. This knowledge is quite useful in sediment studies. The pressure field over a dune changes from very strong adverse pressure gradient to a slightly favorable pressure gradient. Figure 3.22 shows decay of the differences in the bed shear stress versus $\frac{X}{L}$. Peak perturbation versus stress differences in Figures 3.23 and 3.24 shows that the data are well described by a straight line, which could be represented by

$$DU_* = 1.43Du_* - 0.044(Du_*)^2 \qquad (3.4)$$

in which $DU_* = \frac{U_{*c} - U_{*d}}{U_{*c}}$ and $Du_* = \frac{u_{*c} - u_{*d}}{u_{*c}}$

Figure 3.1 Velocity distribution (measured by D. A. Lyn)

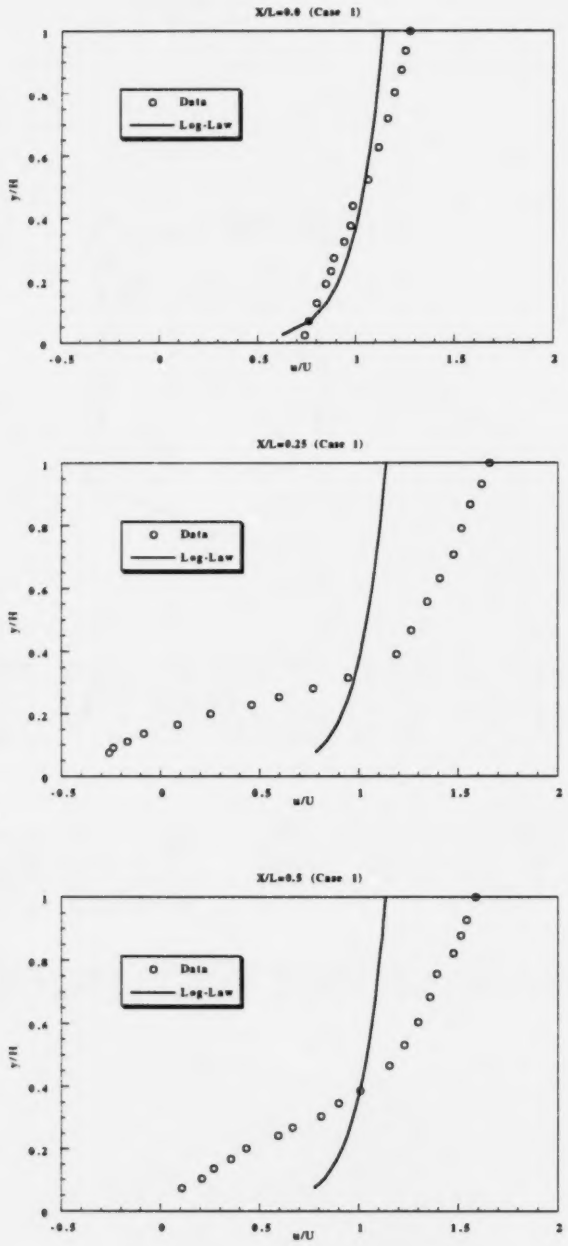


Figure 3.1 (continued)

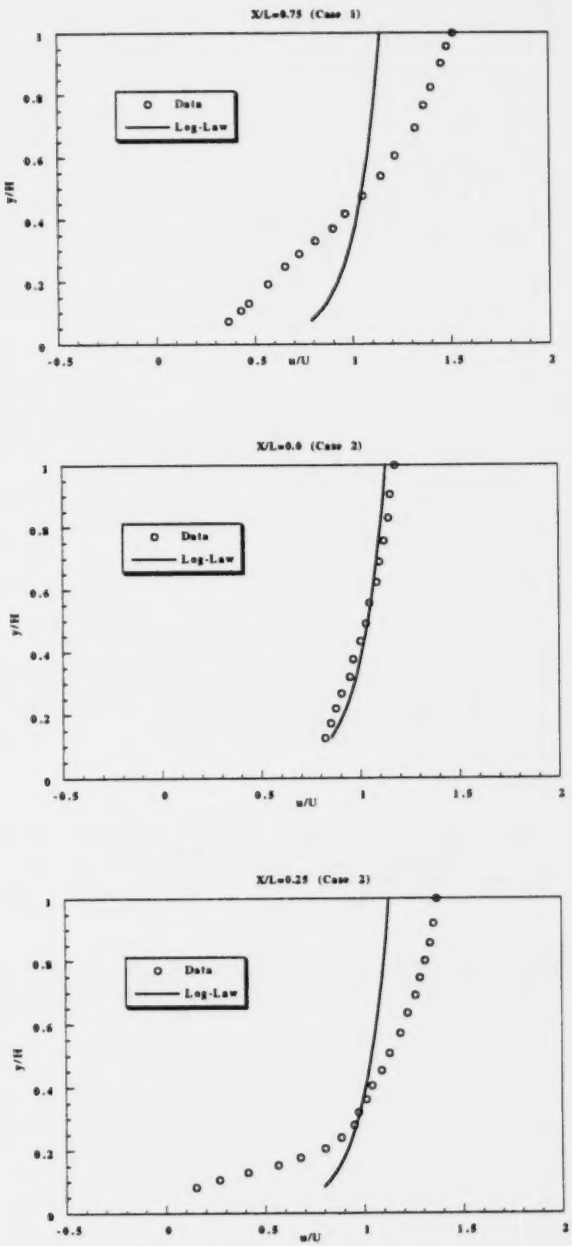


Figure 3.1 (continued)

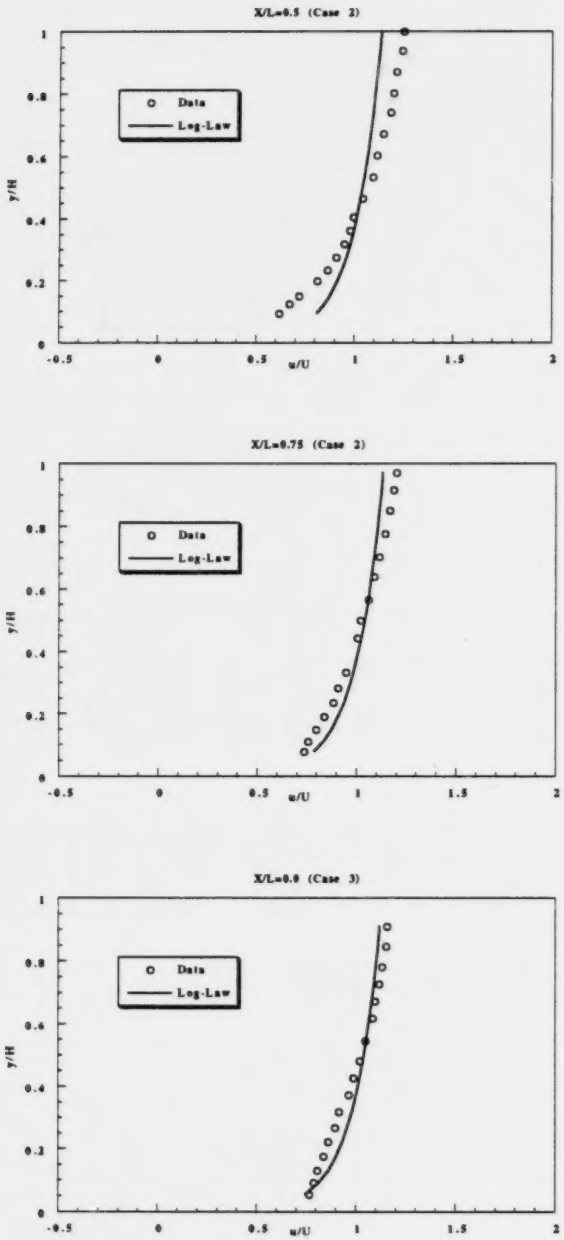


Figure 3.1 (continued)

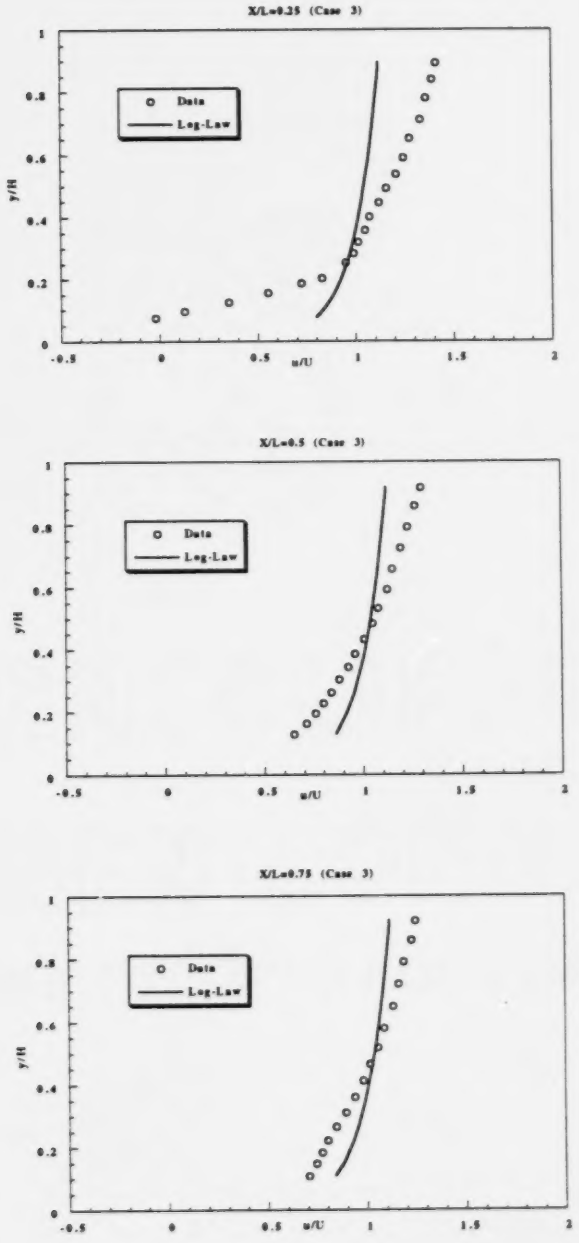


Figure 3.2 Velocity distribution (T5, measured by Delft)

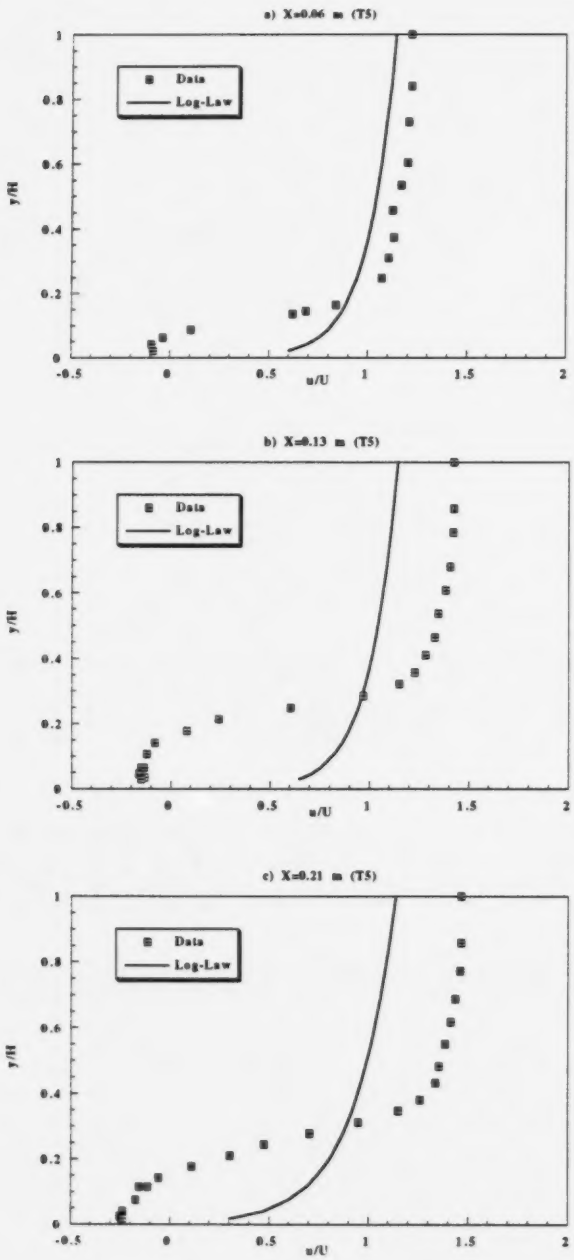


Figure 3.2 (continued)

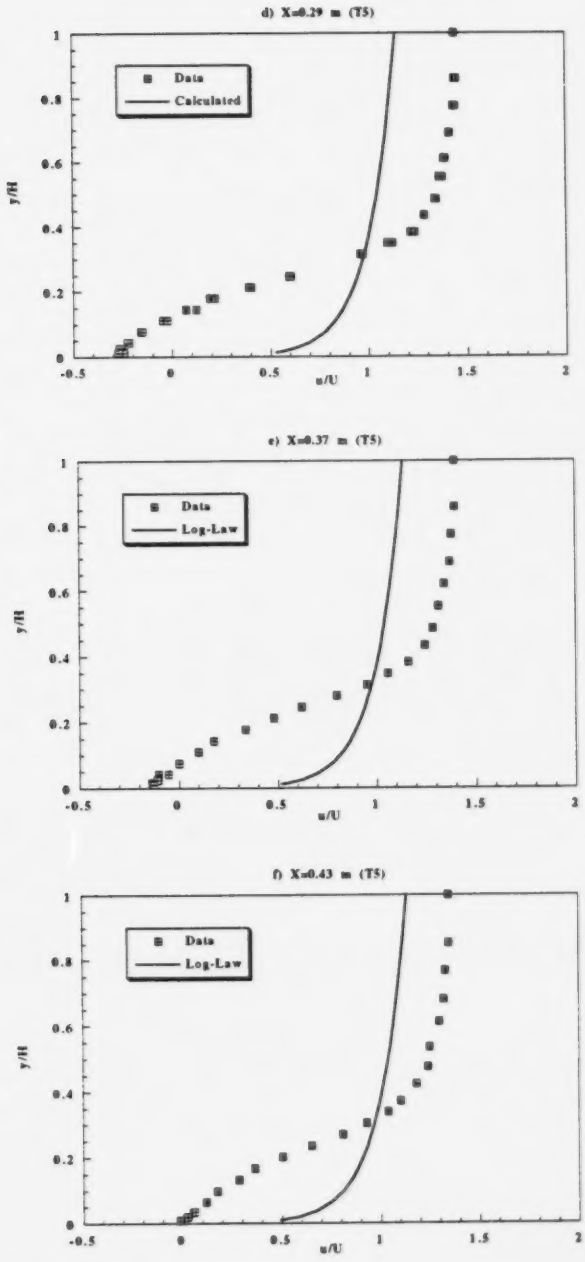


Figure 3.2 (continued)

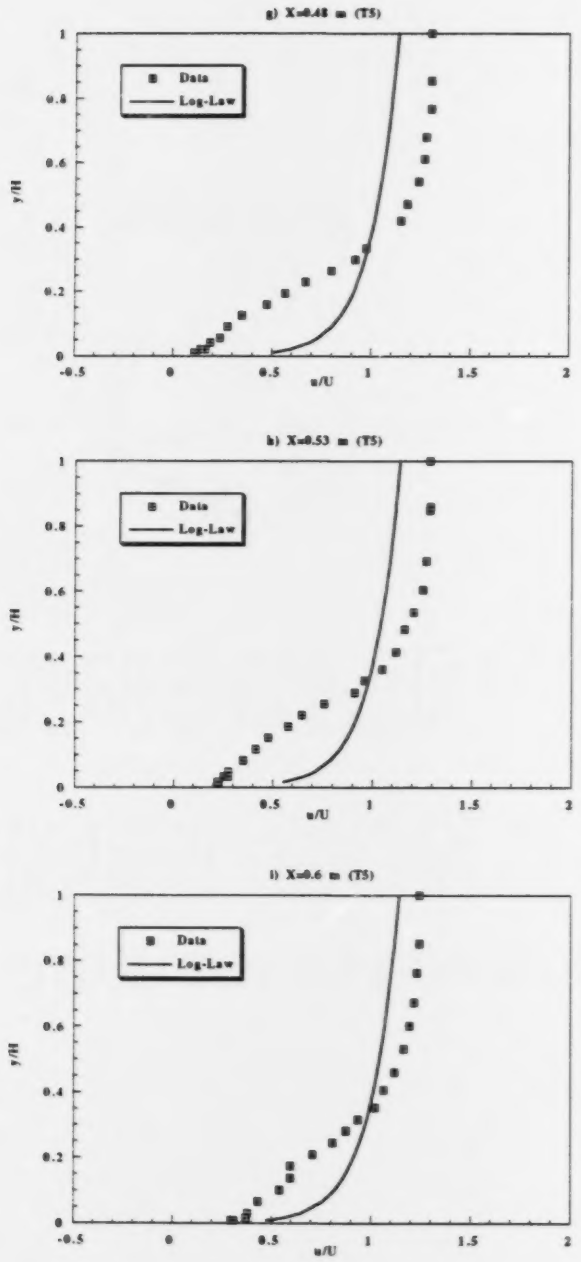


Figure 3.2 (continued)

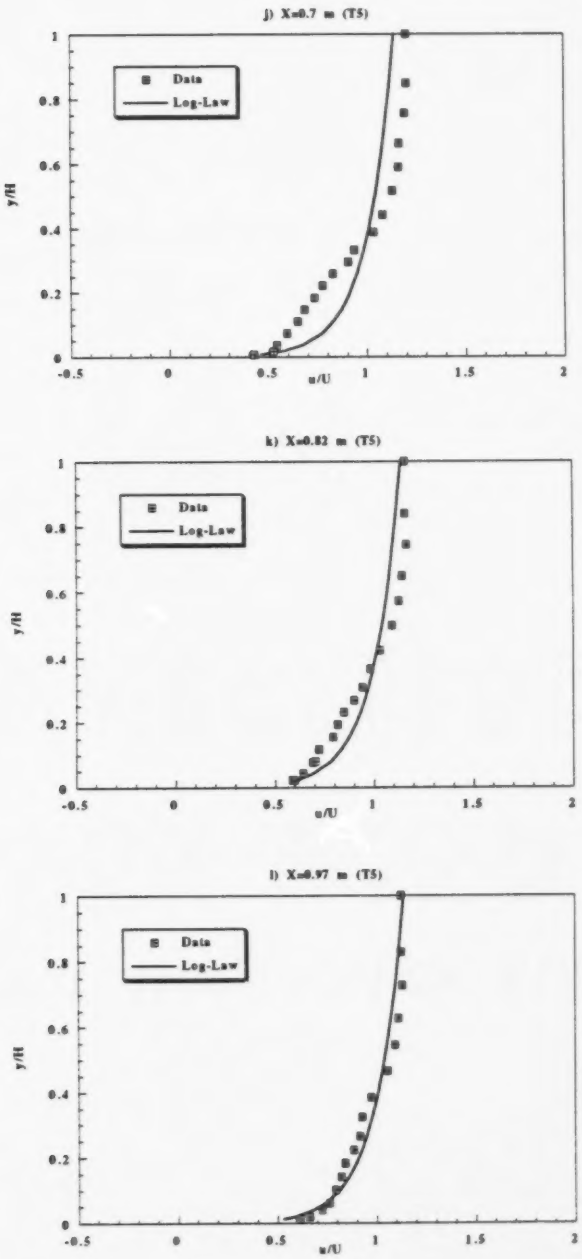


Figure 3.2 (continued)

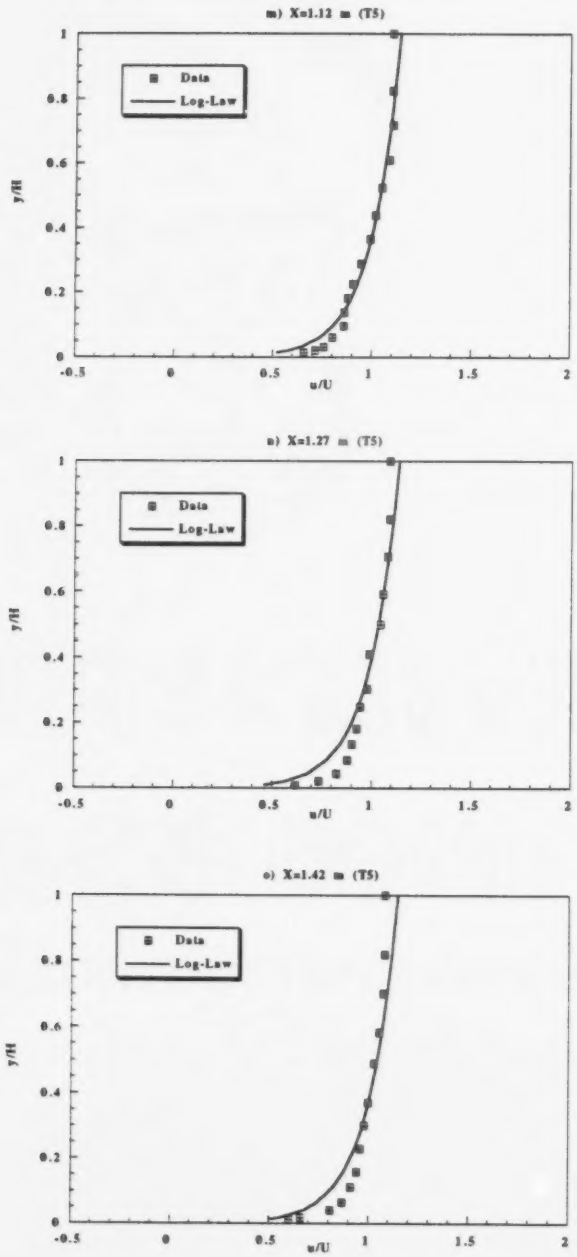


Figure 3.2 (continued)

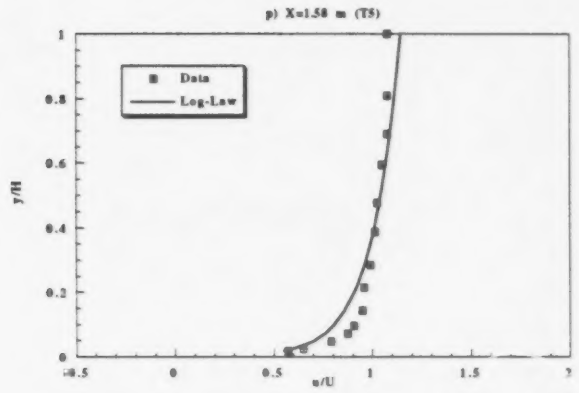


Figure 3.3 Velocity distribution (T6, measured by Delft)

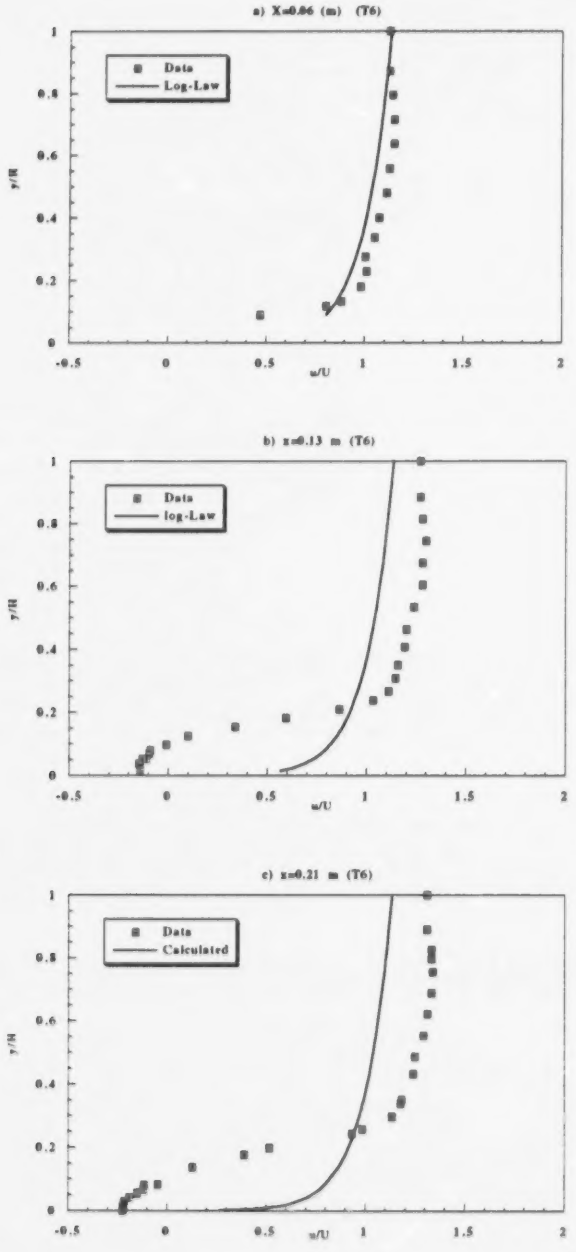


Figure 3.3 (continued)

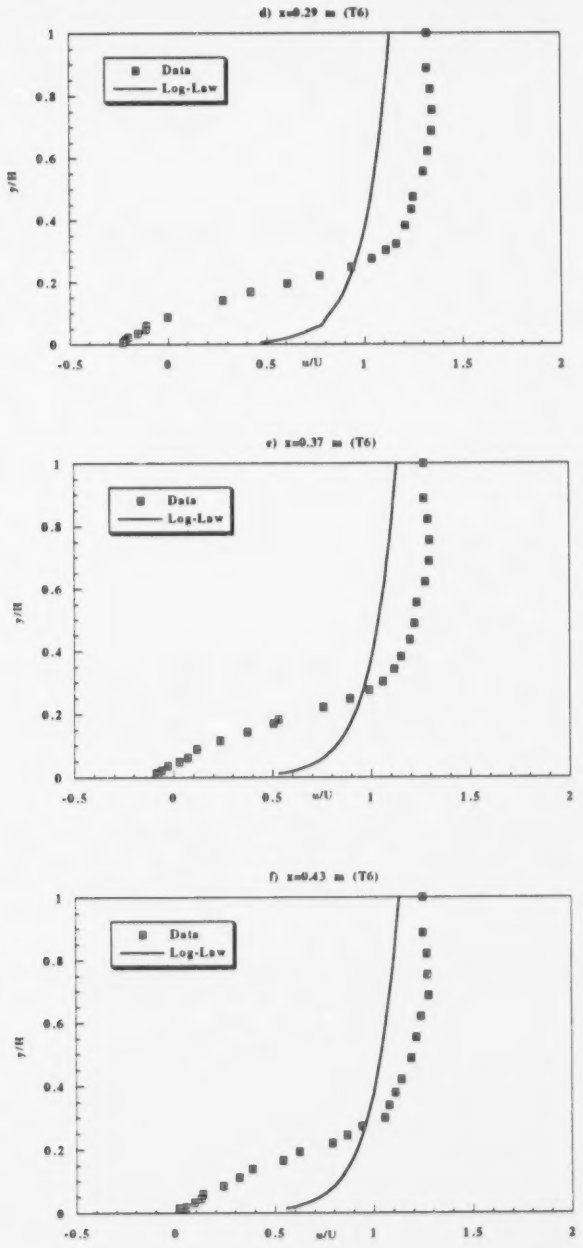


Figure 3.3 (continued)

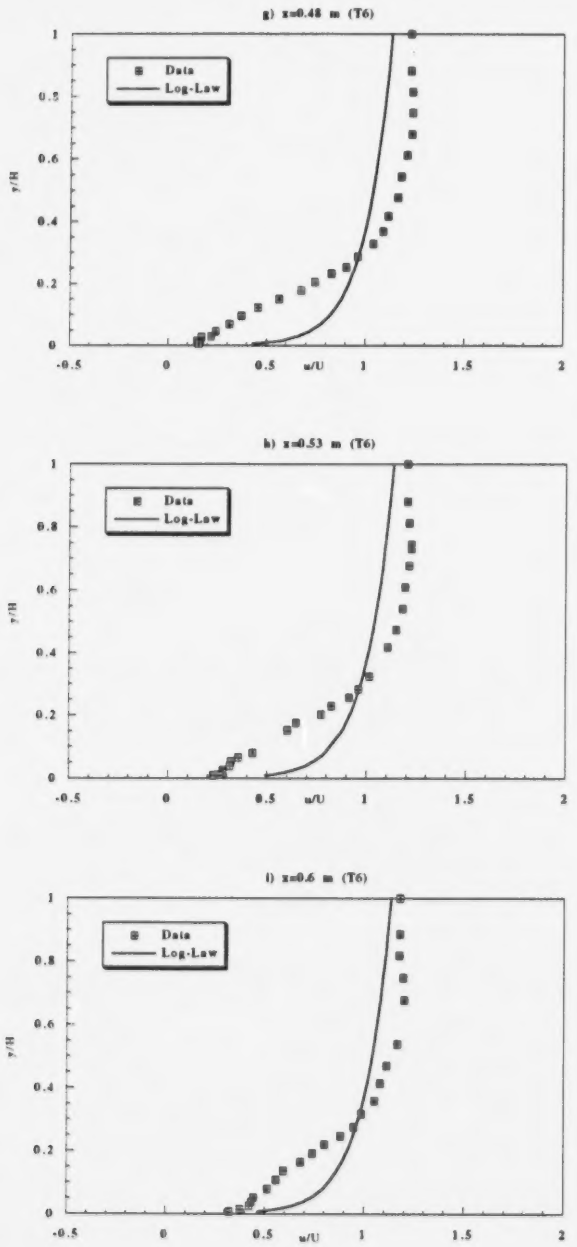


Figure 3.3 (continued)

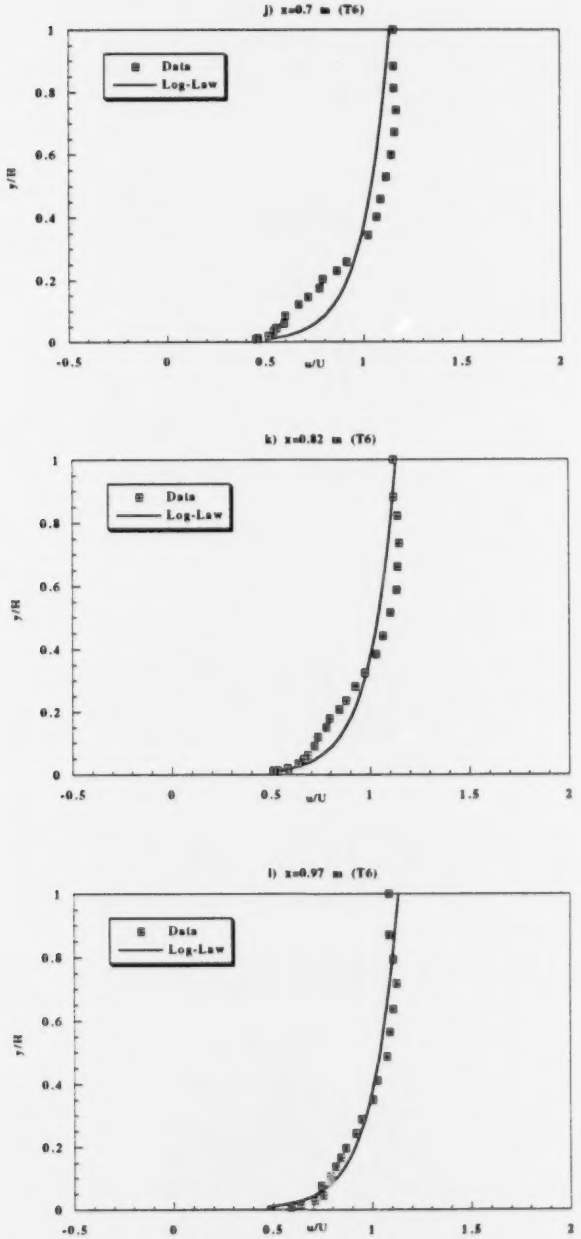


Figure 3.3 (continued)

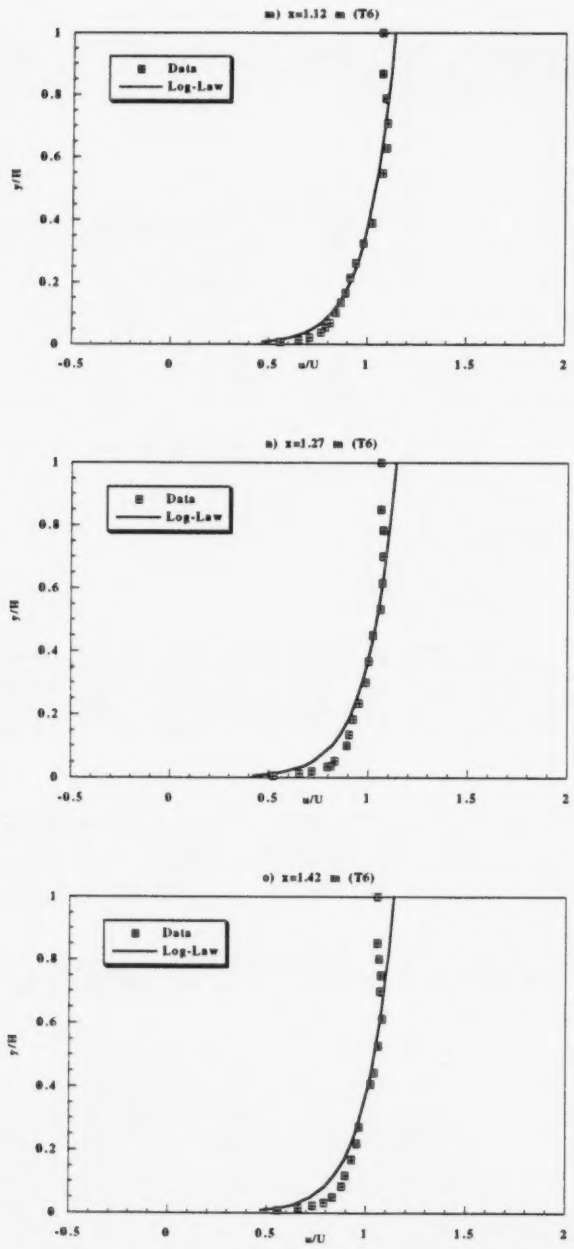
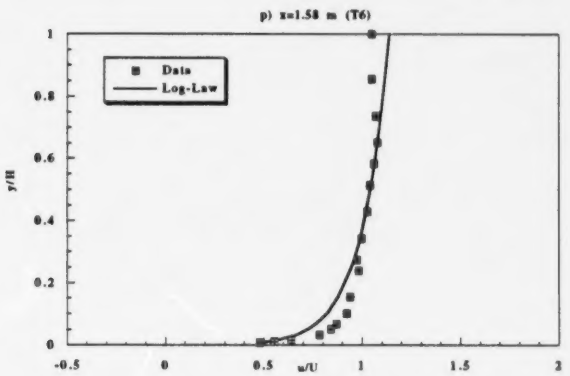


Figure 3.3 (continued)



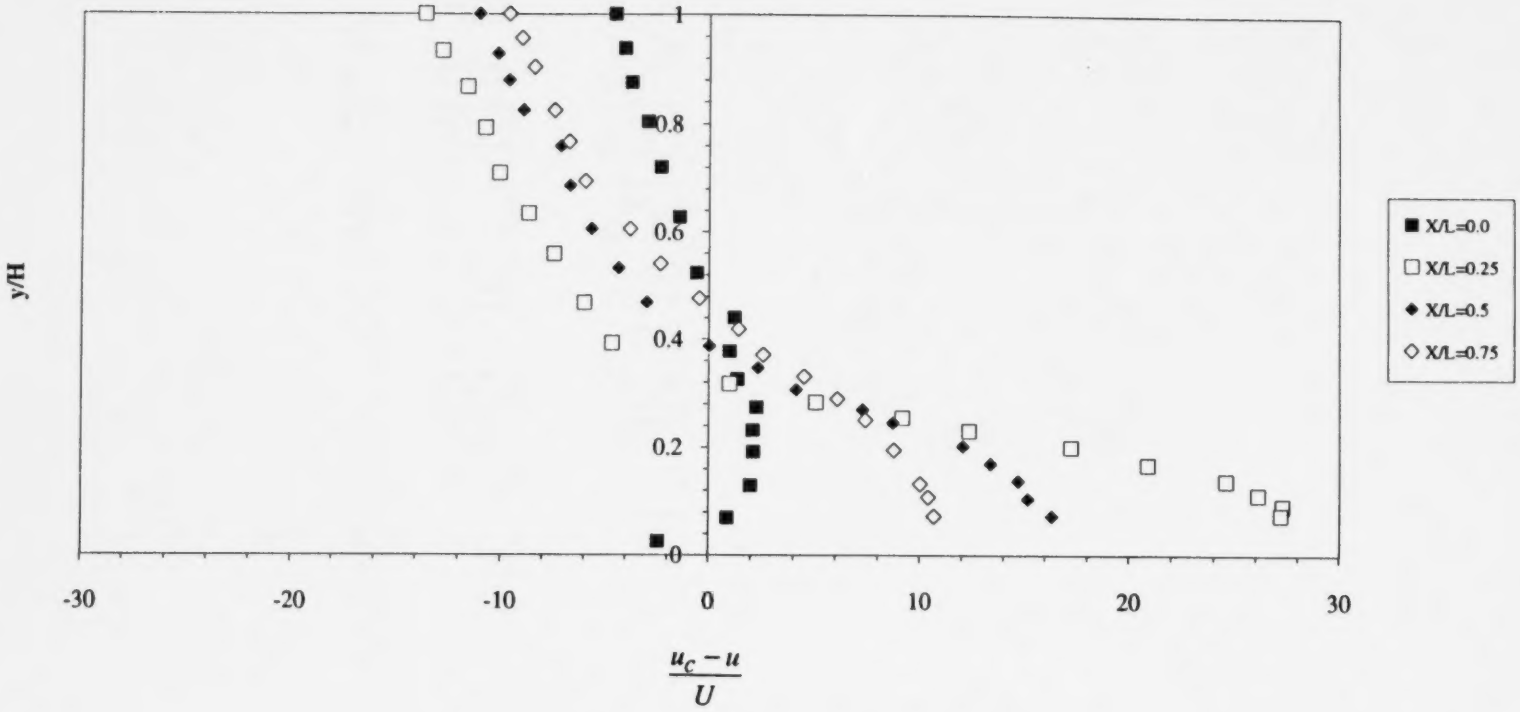


Figure 3.4 Differences in velocity (Case1 measured by D.Lyn)

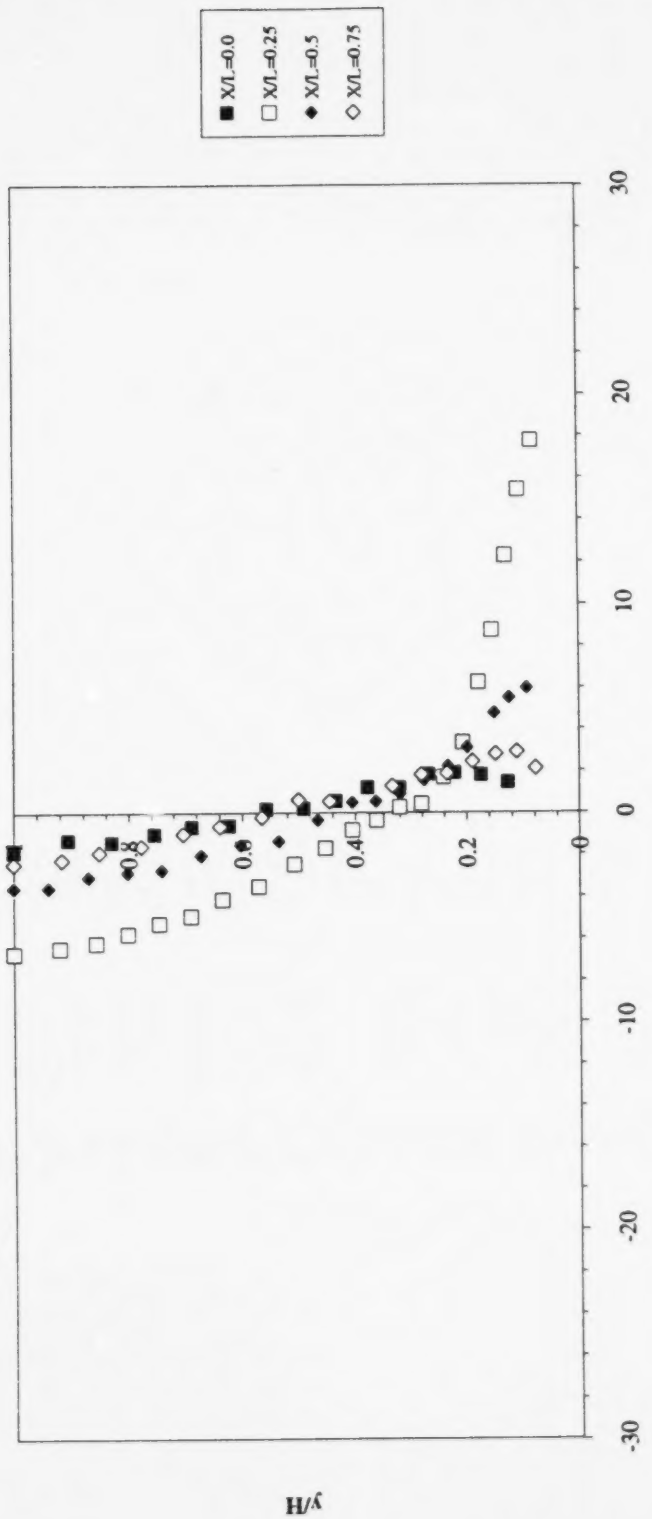


Figure 3.5 Differences in velocity (Case 2 measured by D. Lyn)

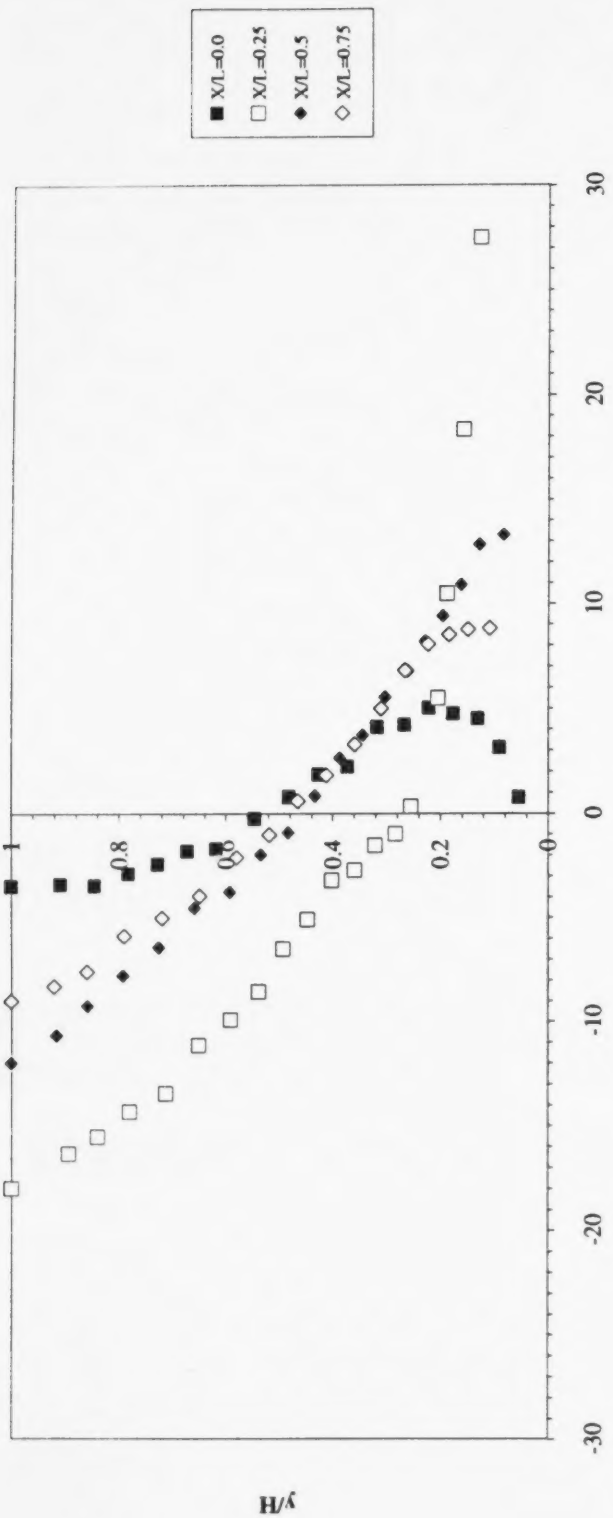


Figure 3.6 Differences in velocity (Case 3 measured by D. Lyn)

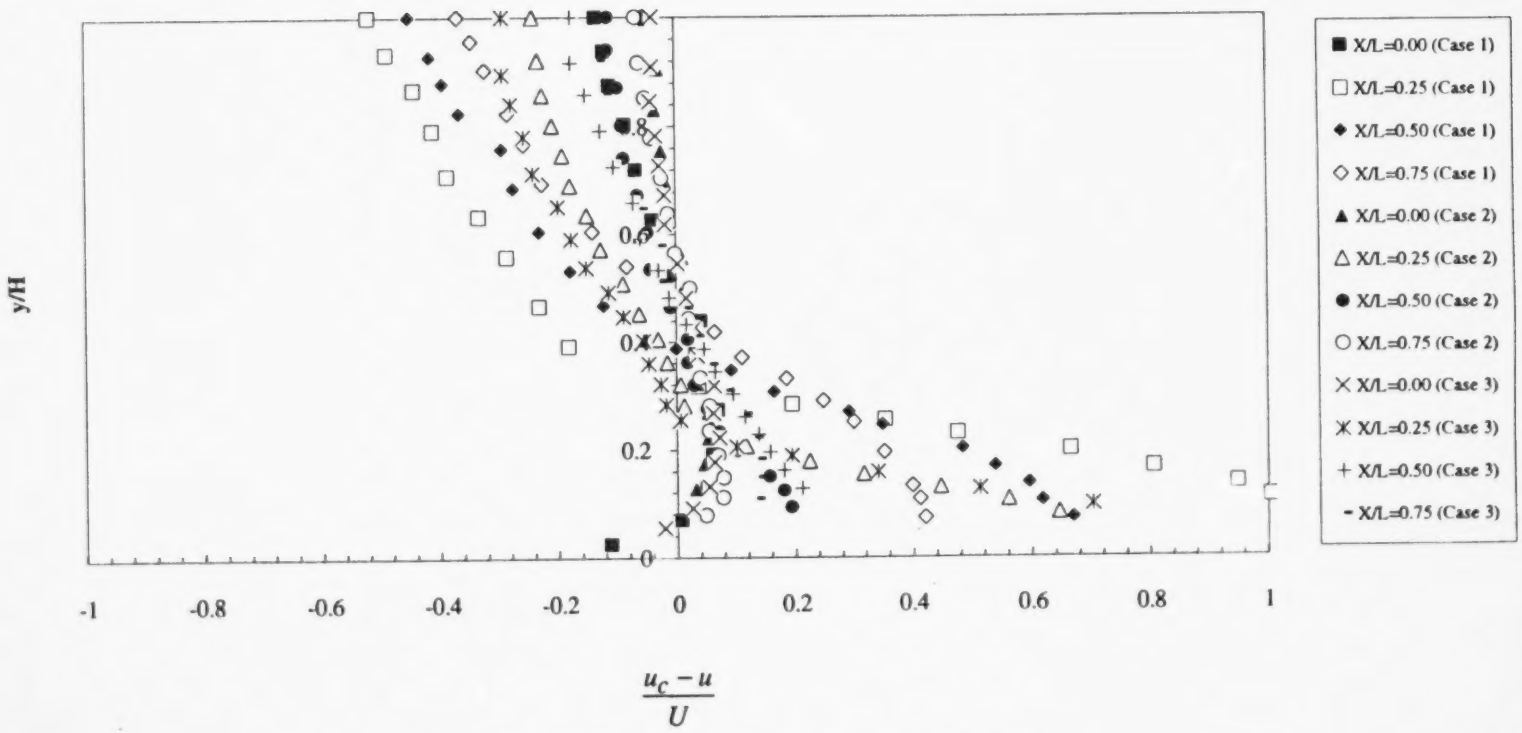


Figure 3.7 Differences in velocity (Case 1.2.3 measured by D. Lyn)

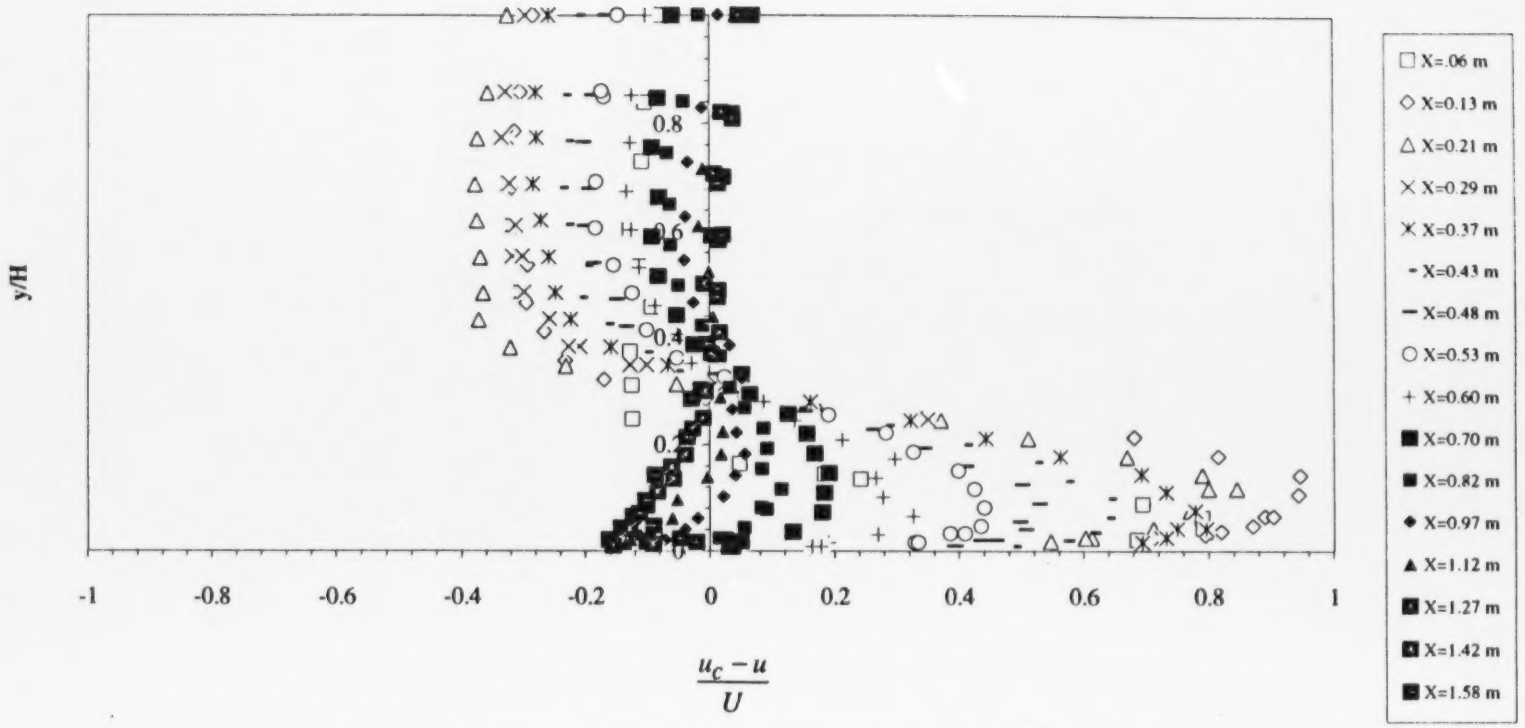


Figure 3.8 Differences in velocity (T5, measured by Delft Hydraulics)

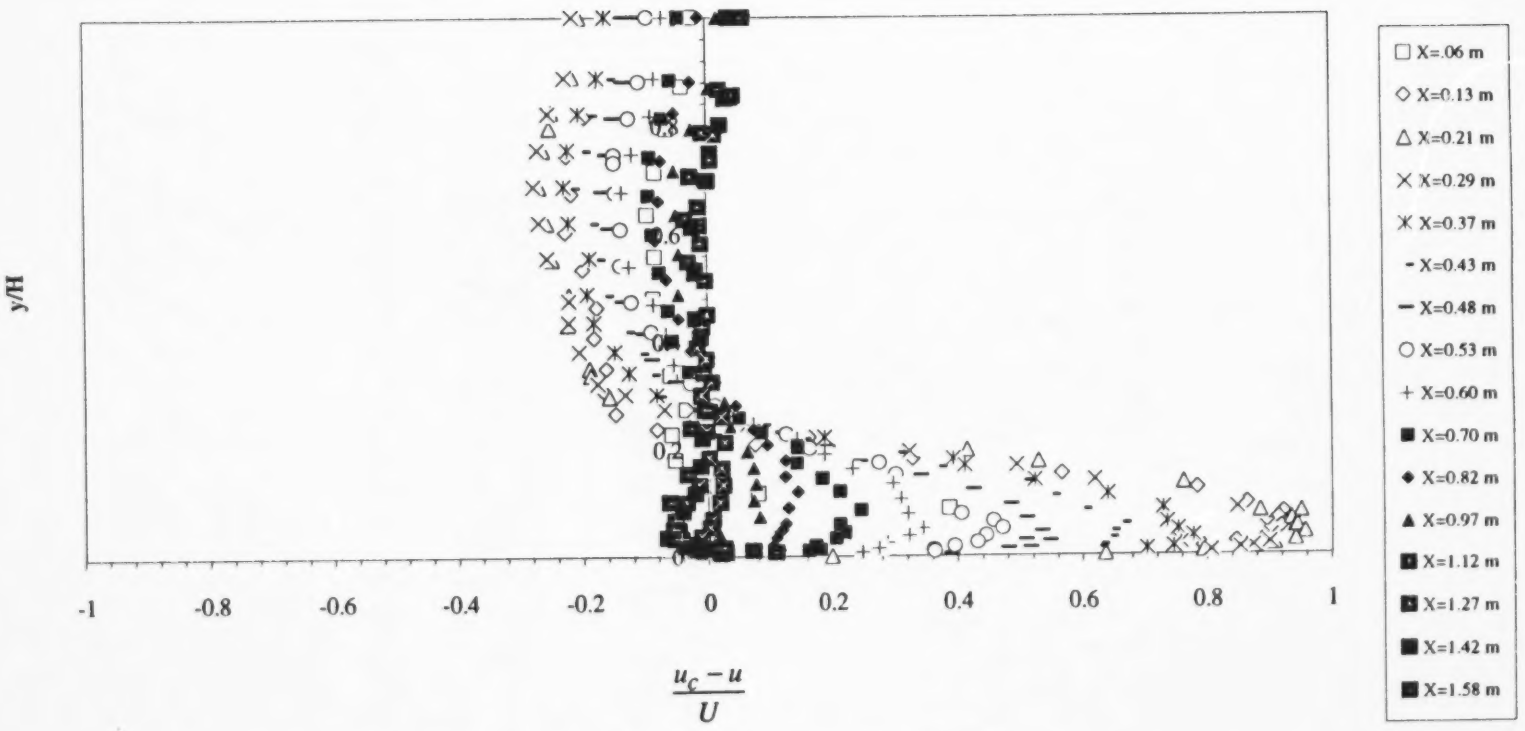


Figure 3.9 Differences in velocity (T6, measured by Delft Hydraulics)

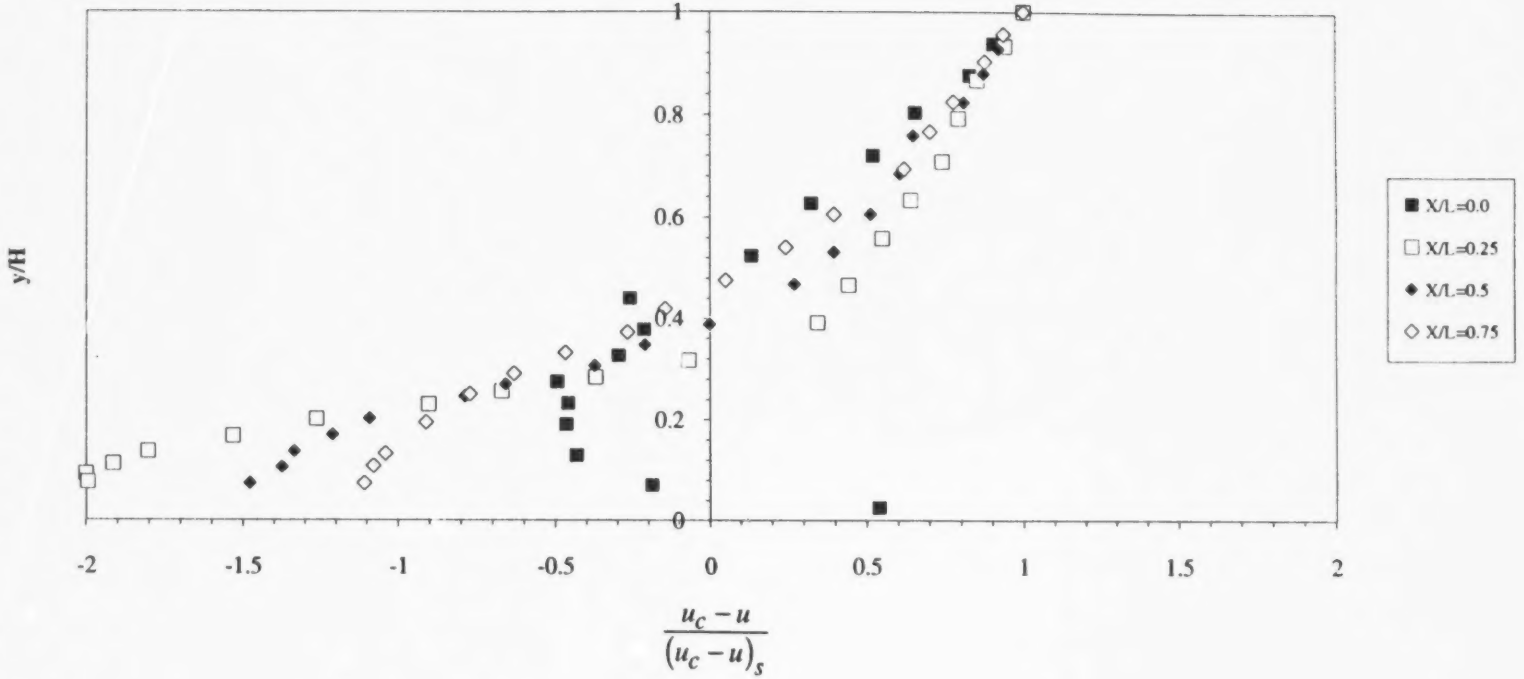


Figure 3.10 Normalized differences in velocity with surface velocity (Case1 measured by D.Lyn)

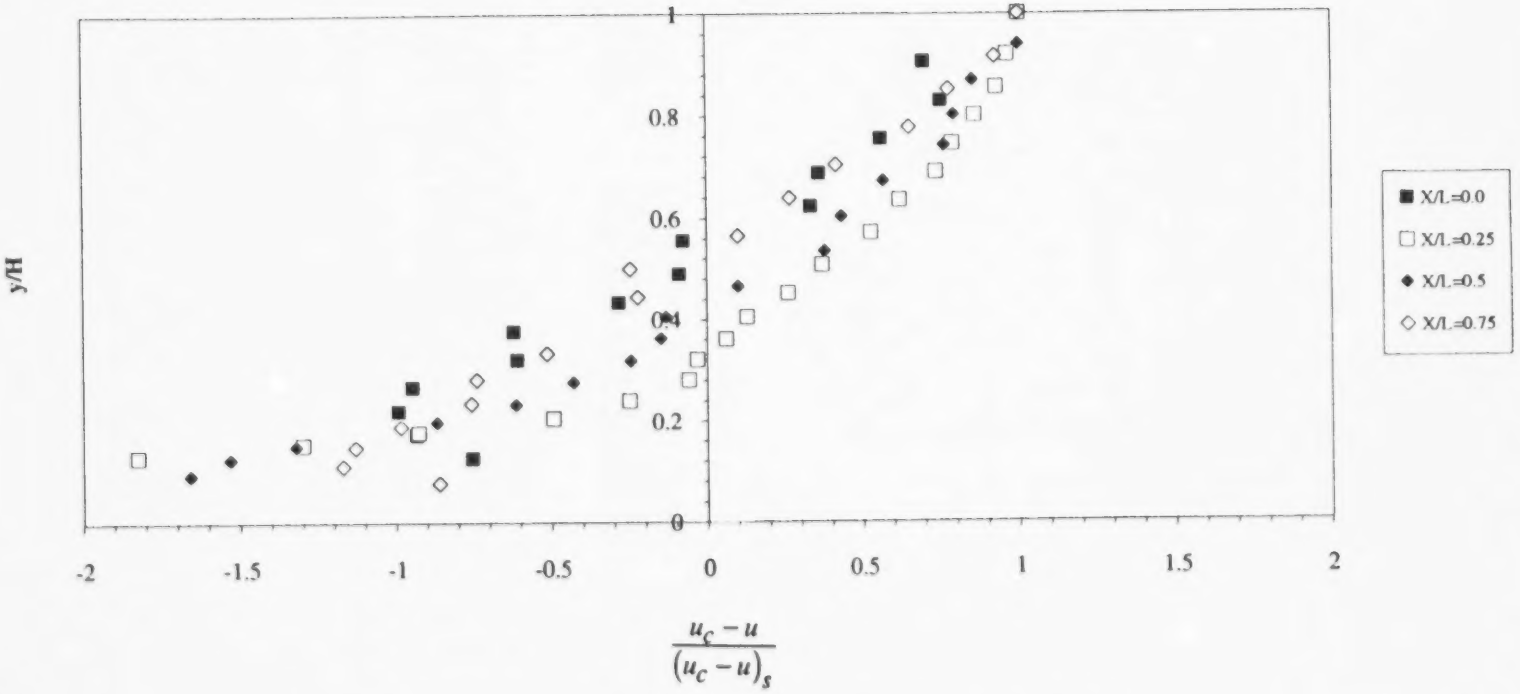


Figure 3.11 Normalized differences in velocity with surface velocity (Case 2 measured by D. Lyn)

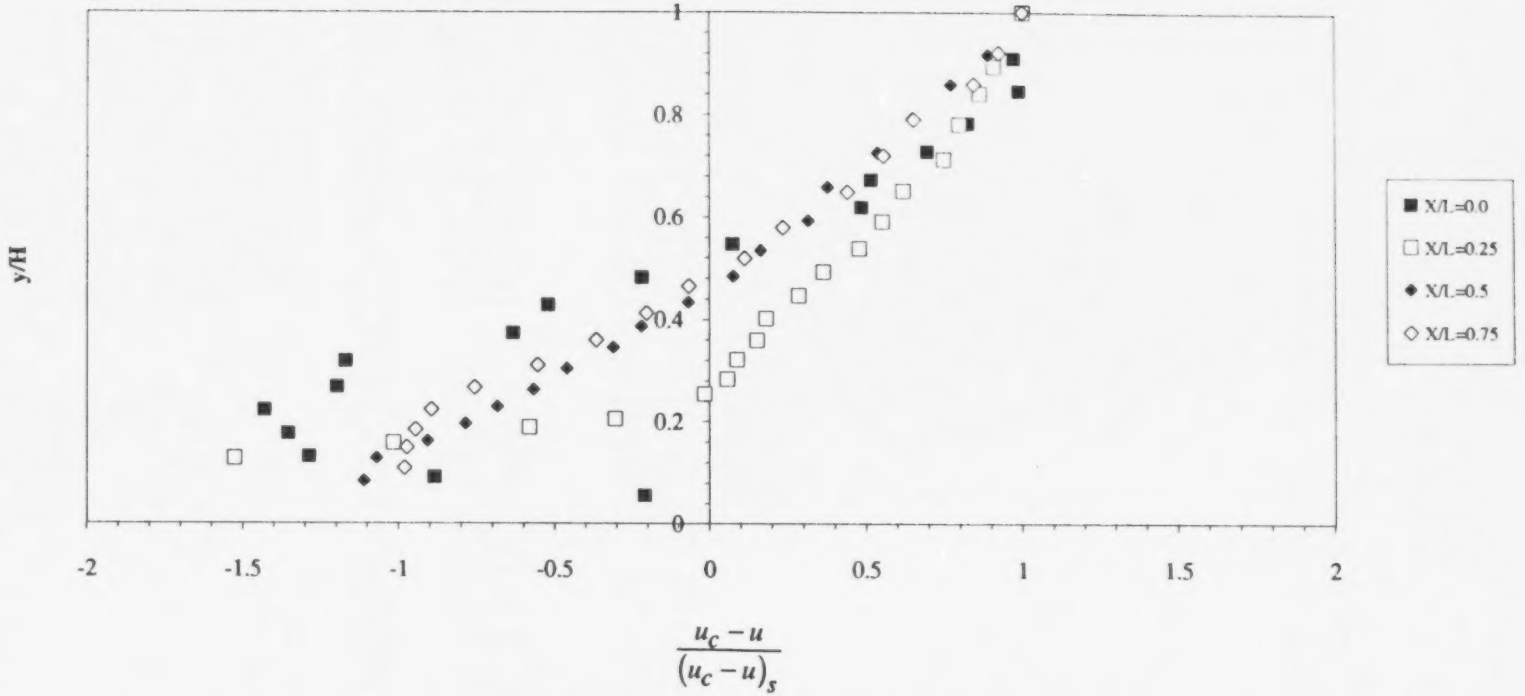


Figure 3.12 Normalized differences in velocity with surface velocity (Case 3 measured by D. Lyn)

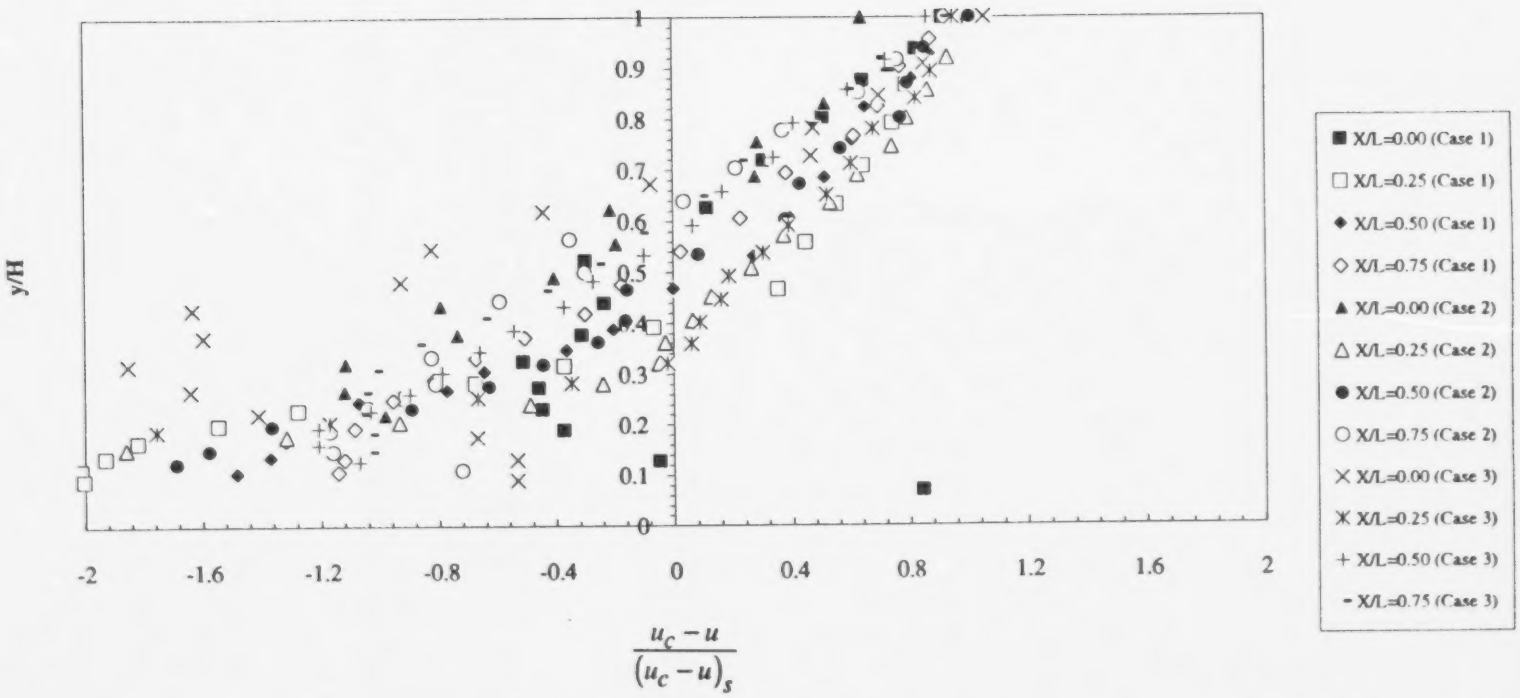


Figure 3.13 Normalized differences in velocity with surface velocity (Case 1.2.3 measured by D. Lyn)

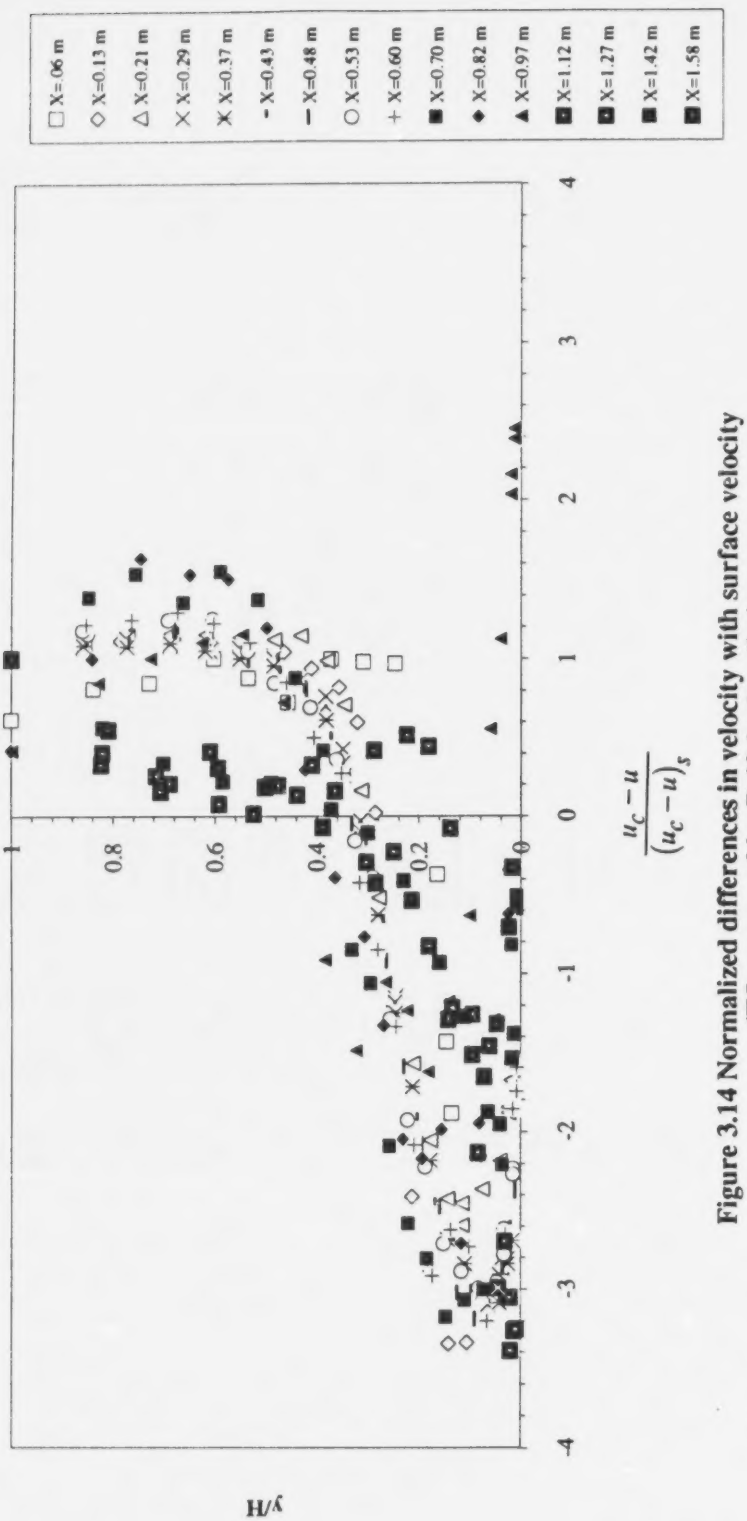


Figure 3.14 Normalized differences in velocity with surface velocity (T5, measured by Delft hydraulics)

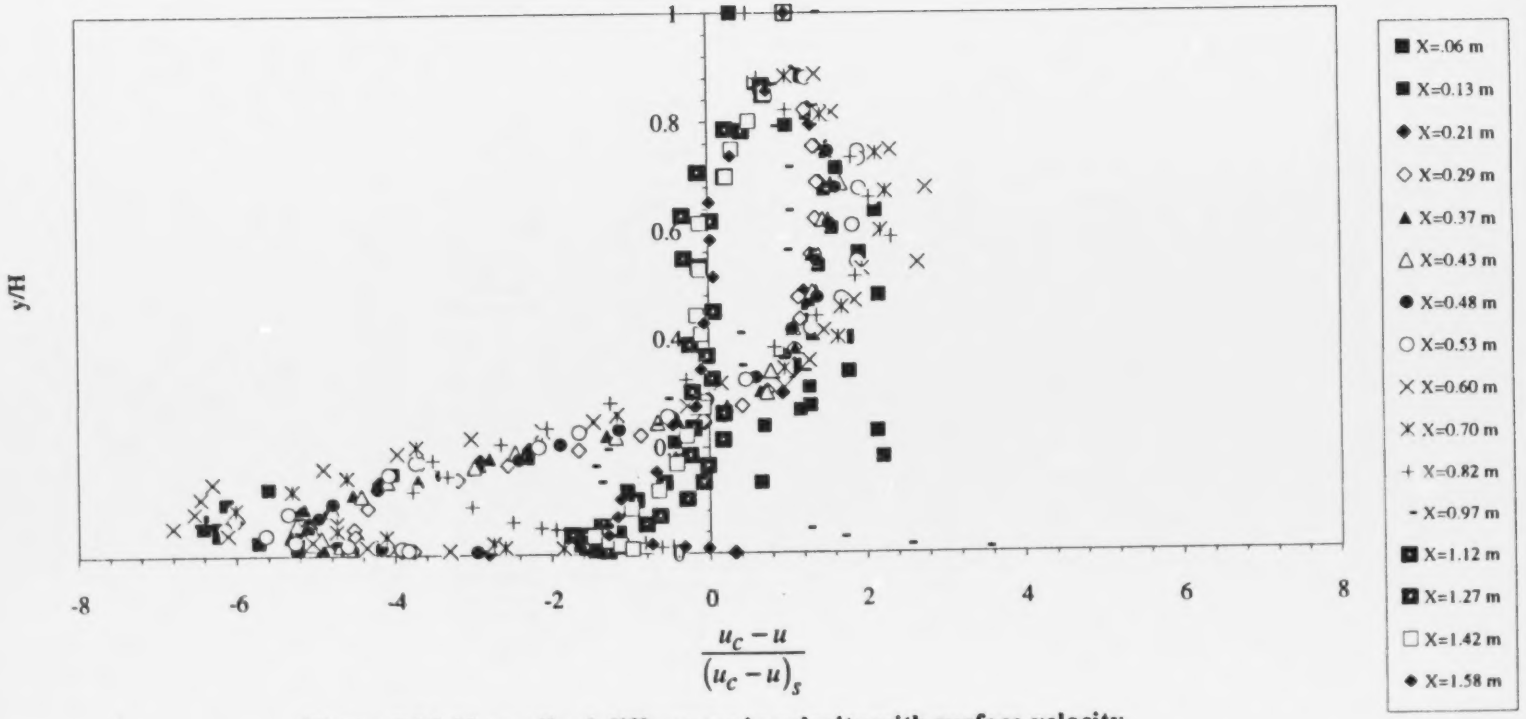


Figure 3.15 Normalized differences in velocity with surface velocity (T6 measured by Delft Hydraulics)

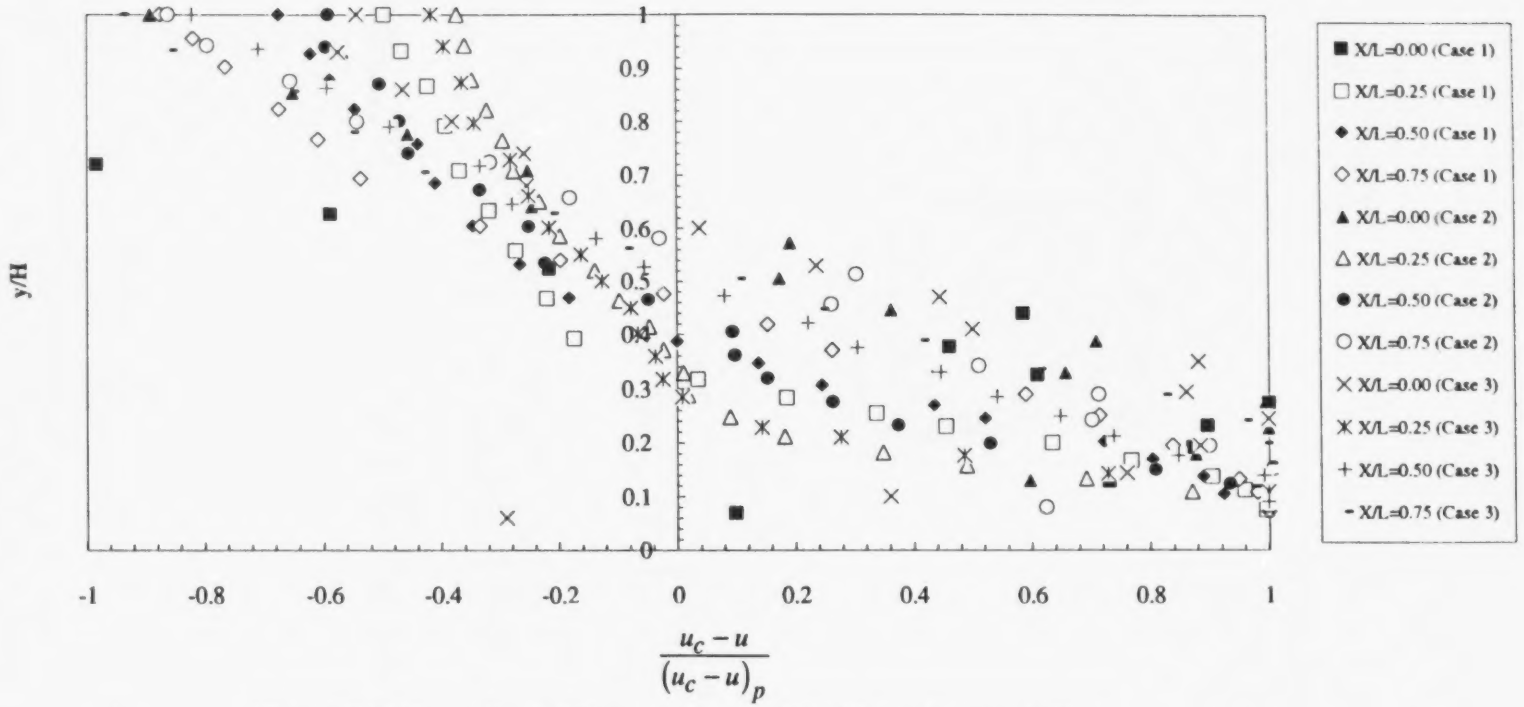


Figure 3.16 Normalized differences in velocity with peak perturbation (measured by D. Lyn)

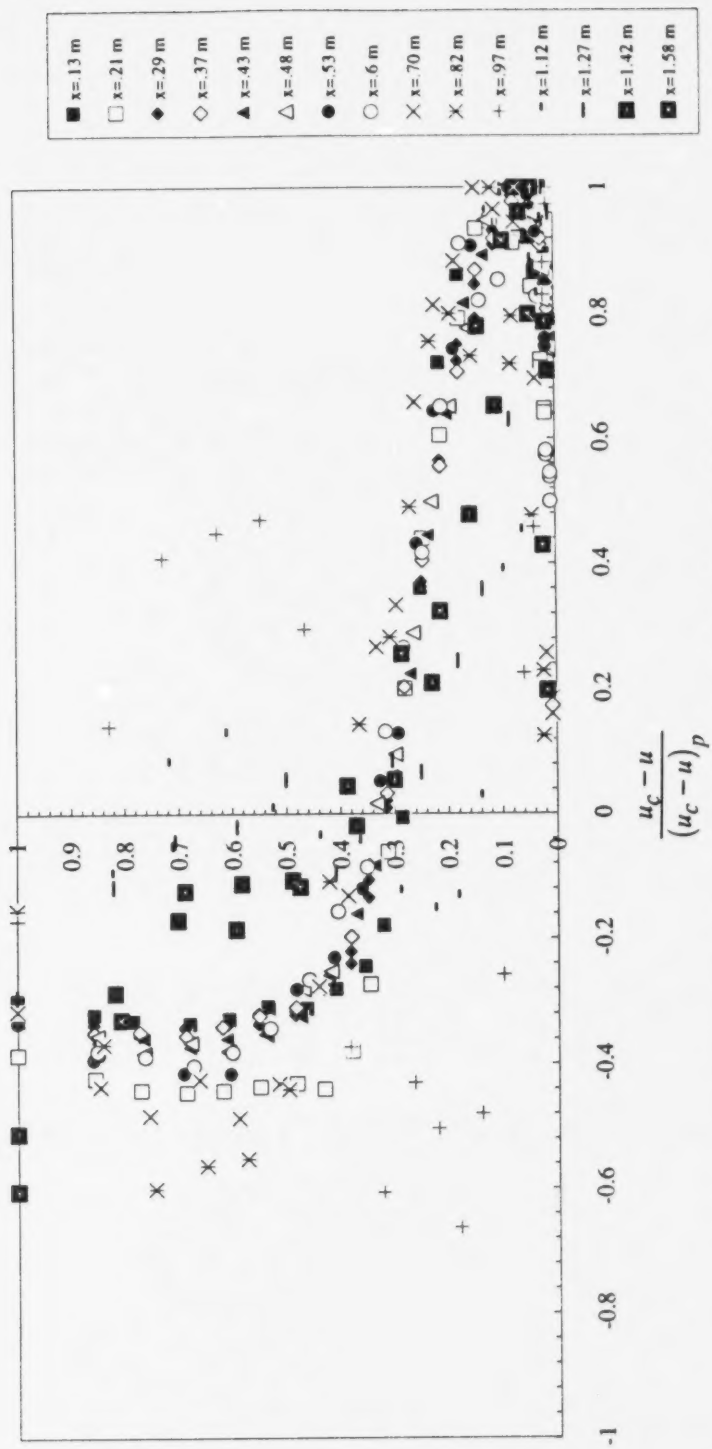


Figure 3.17 Normalized differences in velocity with peak perturbation (T5 measured by Delft Hydraulics)

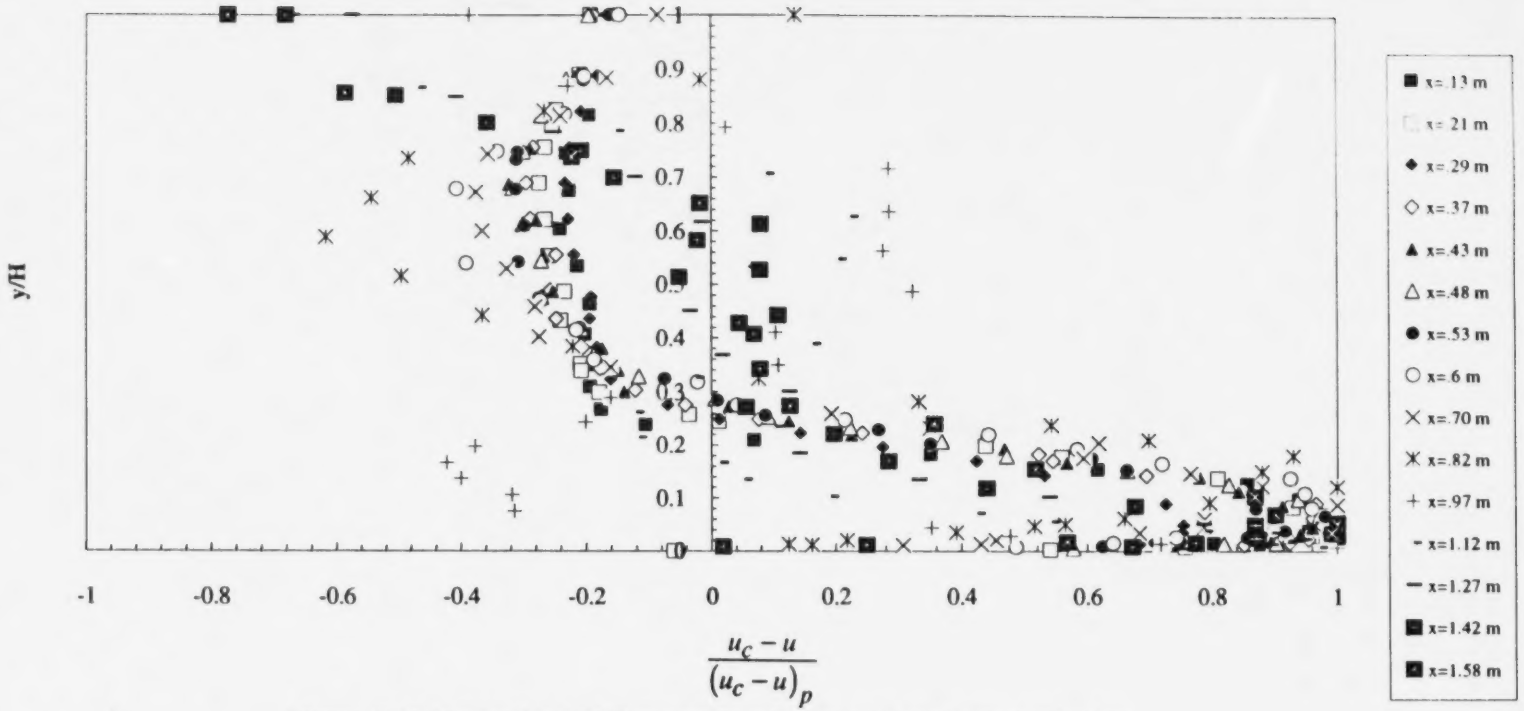


Figure 3.18 Normalized differences in velocity with peak perturbation (T6 measured by Delft Hydraulics)

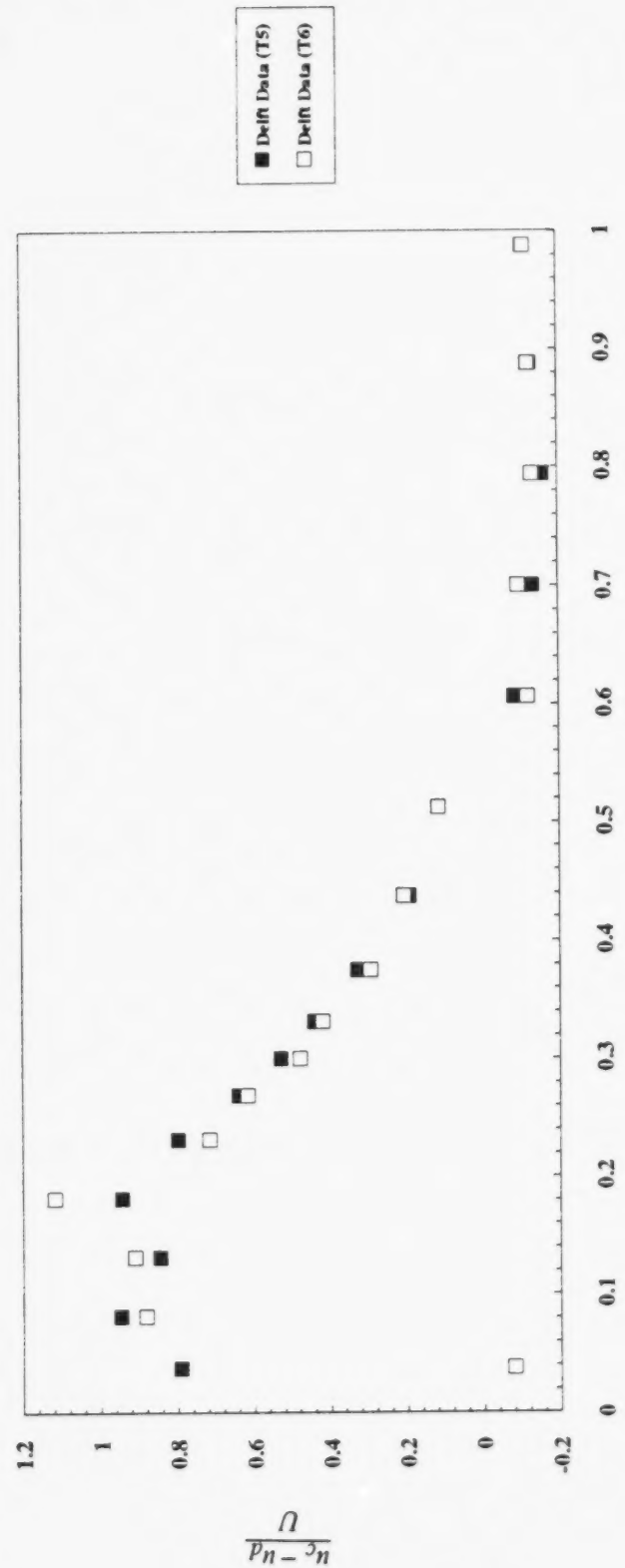


Figure 3.19 Peak perturbation vs wave length

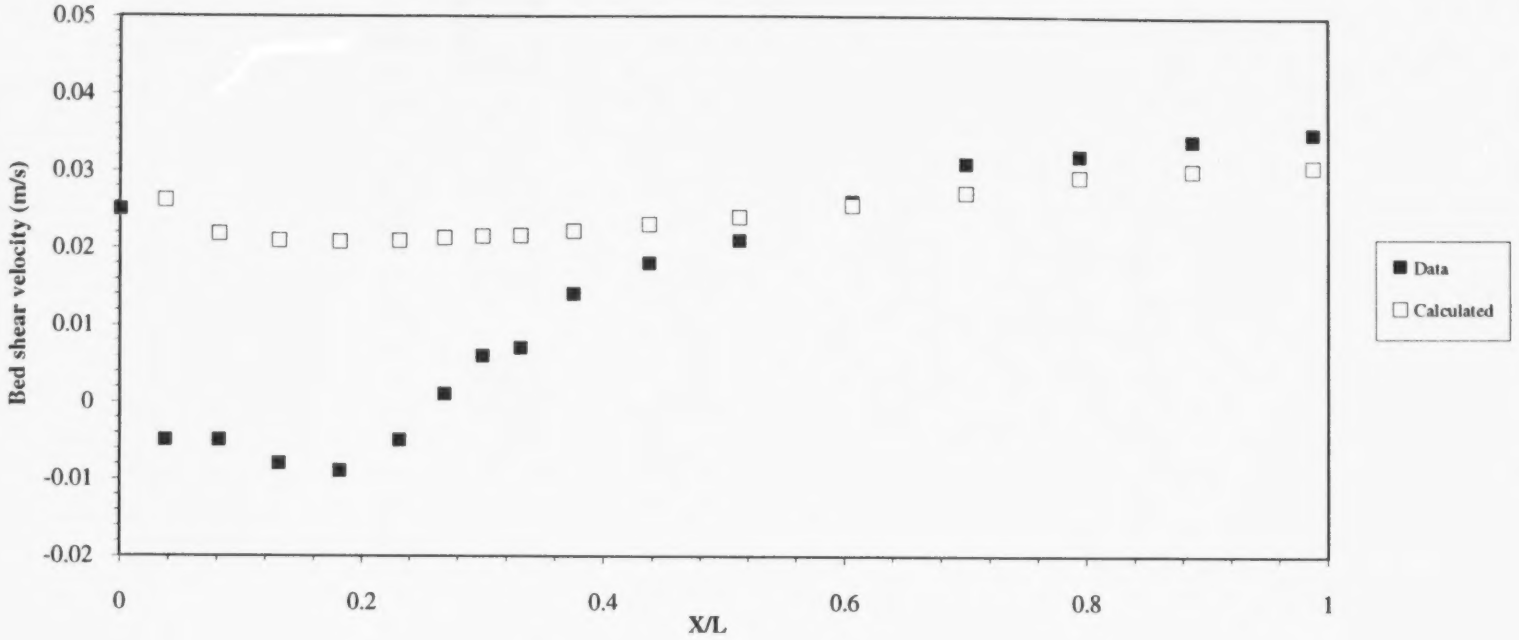


Figure 3.20 Bed shear velocity (T5,measured by Delft Hydraulics)

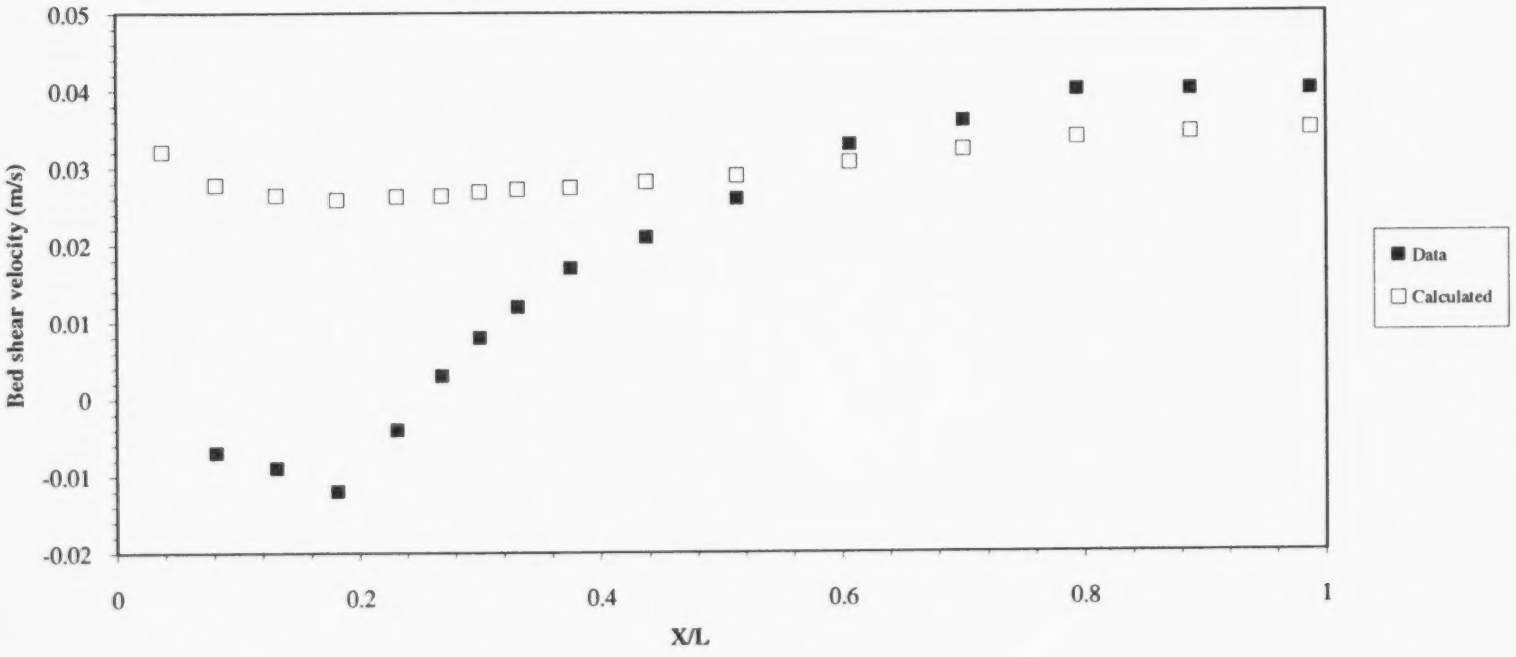


Figure 3.21 Bed shear velocity (T6, measured by Delft Hydraulics)

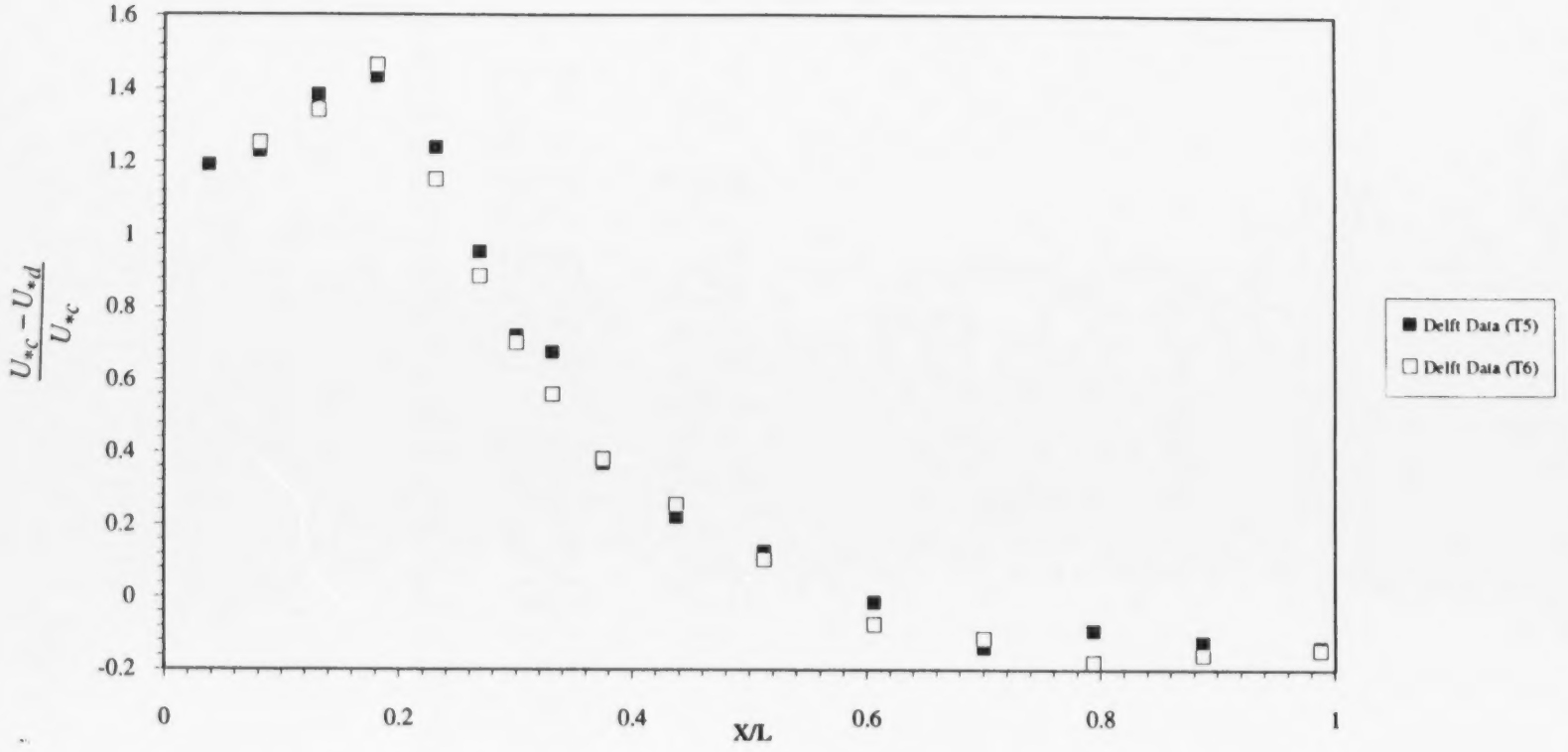


Figure 3.22 Differences in bed shear stress vs. wave length

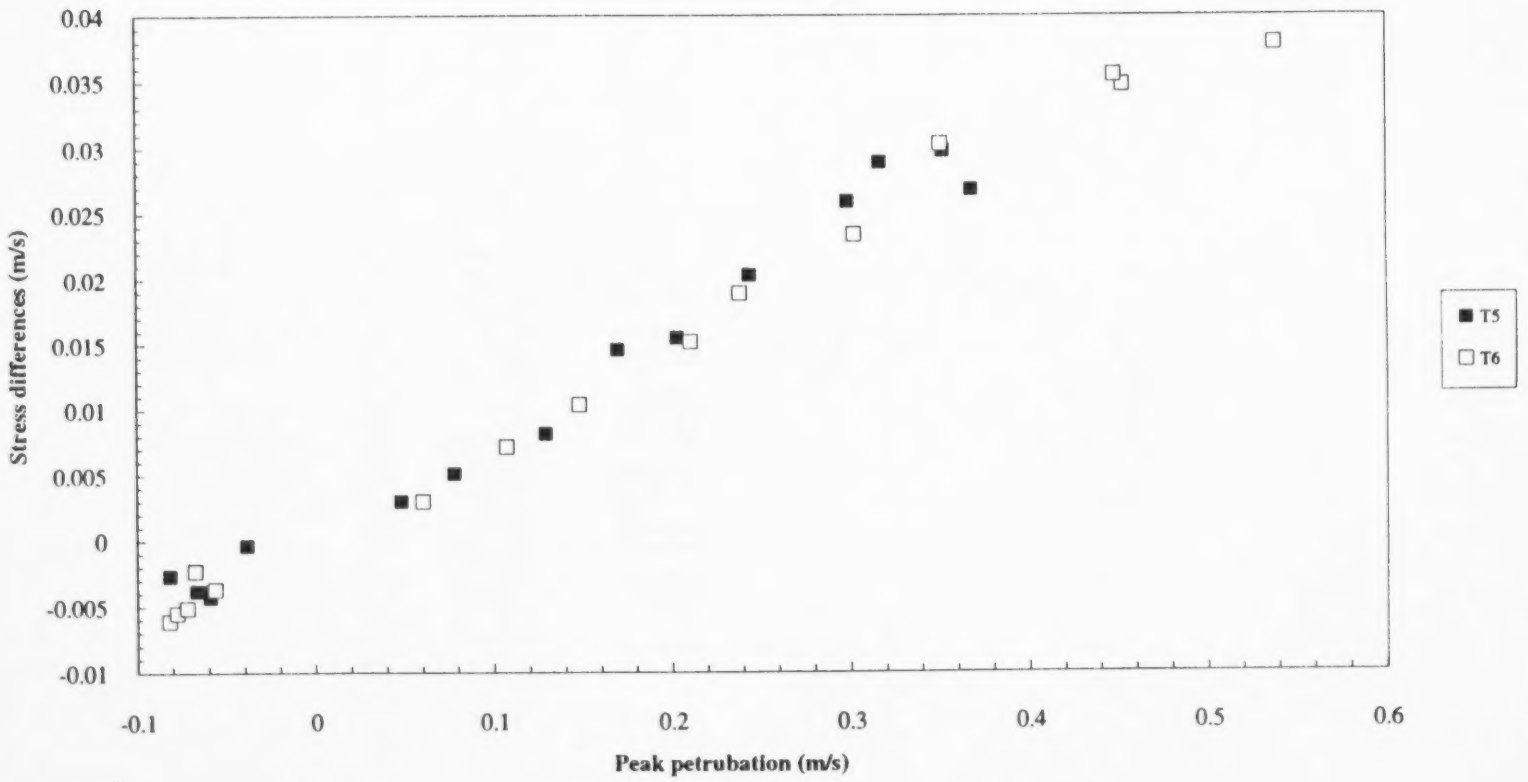


Figure 3.23 Peak velocity vs stress differences

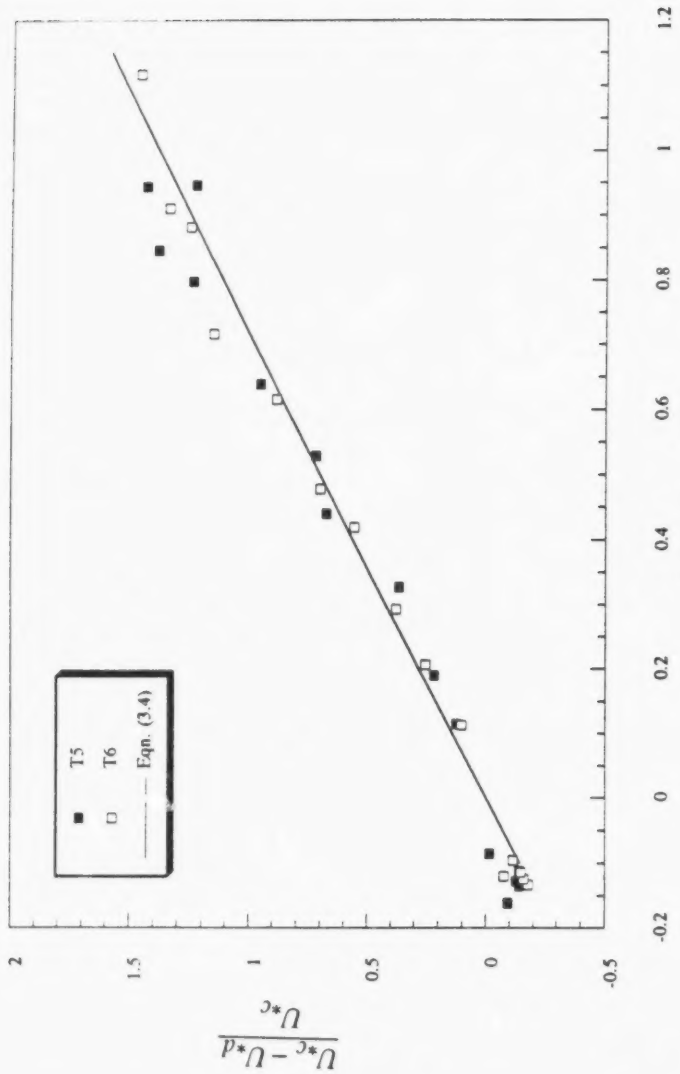


Figure 3.24 Normalized peak perturbation vs stress differences

CHAPTER 4: WALL WAKE ANALYSIS

4.1 Introduction

Two cases of wall-wakes (Lyn, Delft) with favorable pressure gradients were investigated in the present study. These cases are based on the assumption that there exists two distinct regions of flow downstream from the crest of the dune (recirculation region and near-wake region or flow-development region).

The effects of the dune (obstacle) on the velocity distribution by using the wall-wake function and the law of wall are discussed. The present investigation is concerned with developing a relationship comprising the velocity distribution, the size of obstacle and other geometric properties of the channel. The results of the mean velocity profiles, wall shear stress, decay of velocity defect and growth of the half-width of the wake are presented in convenient forms. The basic concern here is the manner in which a wall-wake disturbance develops downstream on a dune.

4.2 Velocity Distribution

Considering the velocity profile in the wake as shown in Figure 4.5, define a velocity defect u_1,

$$u_1 = U_o - u \tag{4.1}$$

where U_o is the maximum (surface velocity).

Let us assume

$$\frac{u_1}{u_{1m}} = f\left(\frac{y}{b}\right) = f(\eta) \tag{4.2}$$

where u_{1m} is the maximum value of u_1 occurring as shown in Figure 4.5, b is the length scale when u_1 is equal to 1/2 of u_{1m}.

A similar expression for τ is

$$\frac{\tau}{\rho u_{1m}^2} = g\left(\frac{y}{b}\right) = g(\eta) \tag{4.3}$$

Because of the effect of vorticity, mixing, back flow in the recirculation region, the shear layer and the wall, the far-wake approximation is not valid. It was shown by Rajaratnam and Rai (1979) that the far-wake approximation might be valid for $\frac{x}{h}$ greater than about 50. Sforza and Mas (1969) studied the flow behind a leading edge obstacle. Their studies were confined to the near-wake region $\frac{x}{h} < 100$, and the flowfield behind the obstacle was assumed to have wakelike characteristics. The author has applied wall-wake characteristics in the present study.

The velocity and length scales, u_{1m} and b for the outer region of the wall were obtained directly from the velocity profile as shown in Figure 4.5. The equation obtained from the wall-wake profile by Schlichting (1968) is as follows.

$$\frac{U_o - u}{u_{1m}} = \left\{1 - 0.293\left(\frac{y}{b}\right)^{3/2}\right\}^2 \tag{4.4}$$

The equation obtained for the shear stress distribution in the "far wake region" is

$$\frac{\tau}{\rho u_{1m}^2} = 0.128\left(\frac{y}{b}\right)\left\{1 - 0.293\left(\frac{y}{b}\right)^{3/2}\right\}^2 \tag{4.5}$$

The following log-law was applied for the inner-region. Figures 4.9 and 4.10 have been used to adjust B in the log-law equation:

$$u^+ = \frac{1}{k}\ln(y^+) + B \qquad (4.6)$$

Herein $u^+ = \frac{u}{u_*}$, $y^+ = \frac{yu_*}{\nu}$ is the dimensionless y-coordinate normalized by the viscous length $\frac{\nu}{u_*}$.

Figures 4.6, 4.7 and 4.8 show that the velocity profiles obtained from experiments over a dune after the recirculation region are approximately similar but do not agree very well with the wall-wake profile (Eqn. 4.4). The reason may be the effect of the near-wake characteristics. The deviation from the wake-profile can be seen to start for T5 at $\frac{y}{b}=0.6$ and for T6 at $\frac{y}{b}=0.7$ and a best-fit line through these points can be described by

$$\frac{U_0 - u}{u_{1m}} = 0.467 - 0.61\ln\frac{y}{b} \qquad (4.7)$$

The velocity distribution for the inner region is plotted in Figures 4.9 and 4.10 . The following equation from the log-law was found to be the best-fit line for our data.

$$\frac{u}{u_*} = 5.75\ log\frac{y}{k_s} + 8.5 \qquad (4.8)$$

The power law form for the velocity profile for the inner region can be obtained from Figures 4.11 and 4.12. The following equation was obtained from the plots.

$$\frac{u}{u_*} = 18\left(\frac{y}{b}\right)^{1/5} \qquad (4.9)$$

The value of the coefficient ,18, is different from that obtained by Rajaratnam and Rai (1979) for the far-wall wake which it was 28.

$\frac{u_{1m}}{U_o}$ versus $\frac{x}{h}$ is plotted in Figure 4.13. The following equation which shows the velocity defect decay for the wall-wake is very different from the best-fit line from the present data.

$$\frac{u_{1m}}{U_o} = 1.3\left(\frac{x}{C_D h}\right)^{-1/2} \qquad \text{for } \frac{x}{h}>5.2 \qquad (4.10)$$

where C_D is the drag coefficient.

The growth of the half-width (b) of the wake is plotted against x/h in Figure 4.14. Due to the slope of the bed the best fit line is almost constant and equal to *10h*.

4.3 Bed Shear stress

The present study just approaches to after recirculation region through the wall-wake characteristics.

$\frac{\tau_c - \tau_o}{0.5\rho u_{1m}^2}$ versus $\frac{x}{h}$ is plotted in Figure 4.15 . It does not follow the equation obtained for the wall-wake. The bed shear stress distribution on a dune and on a flat bed is very different.

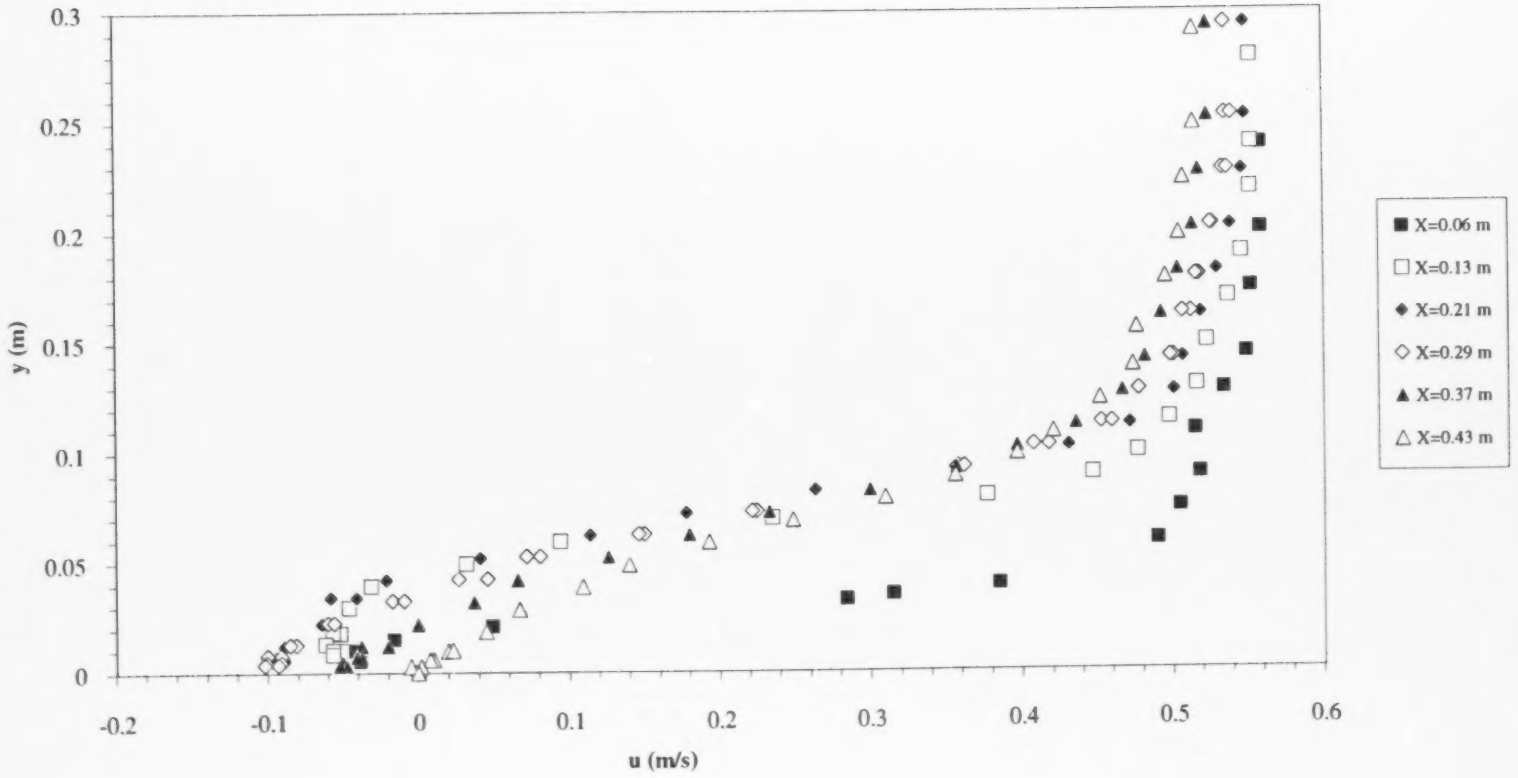


Figure 4.1 Velocity distribution in the region of seperation (T5)

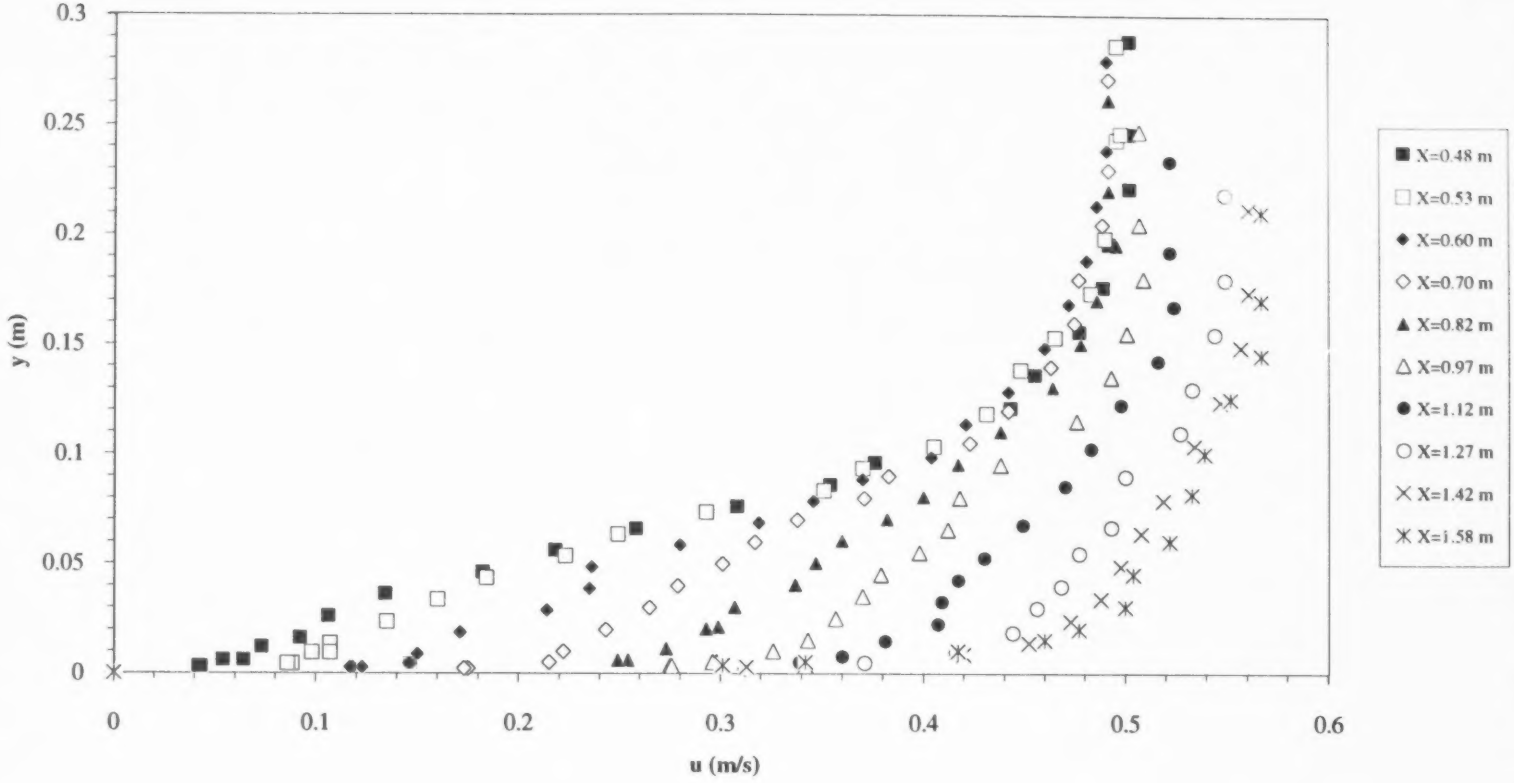


Figure 4.2 Velocity distribution in the wake region (T5)

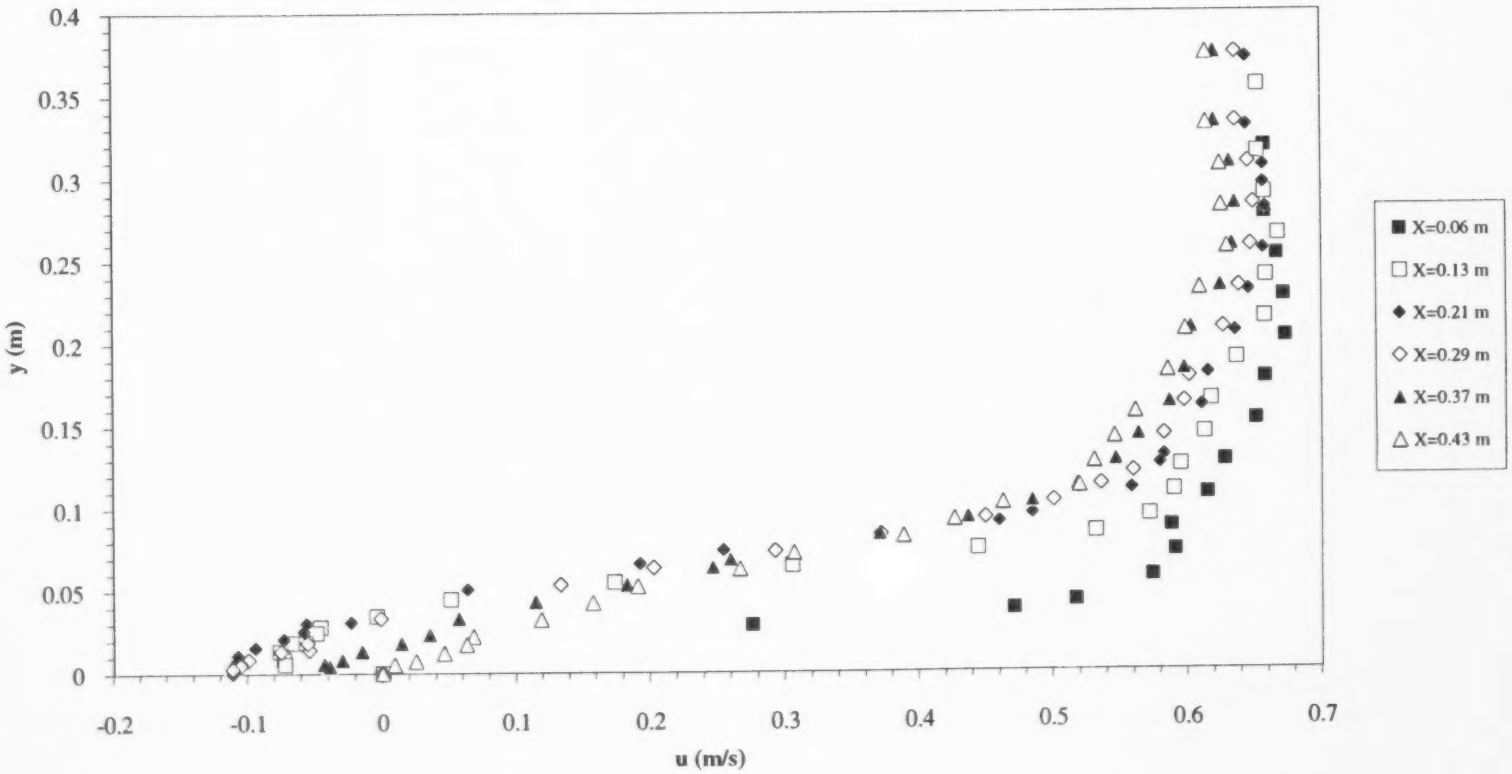


Figure 4.3 Velocity distribution in the region of seperation (T6)

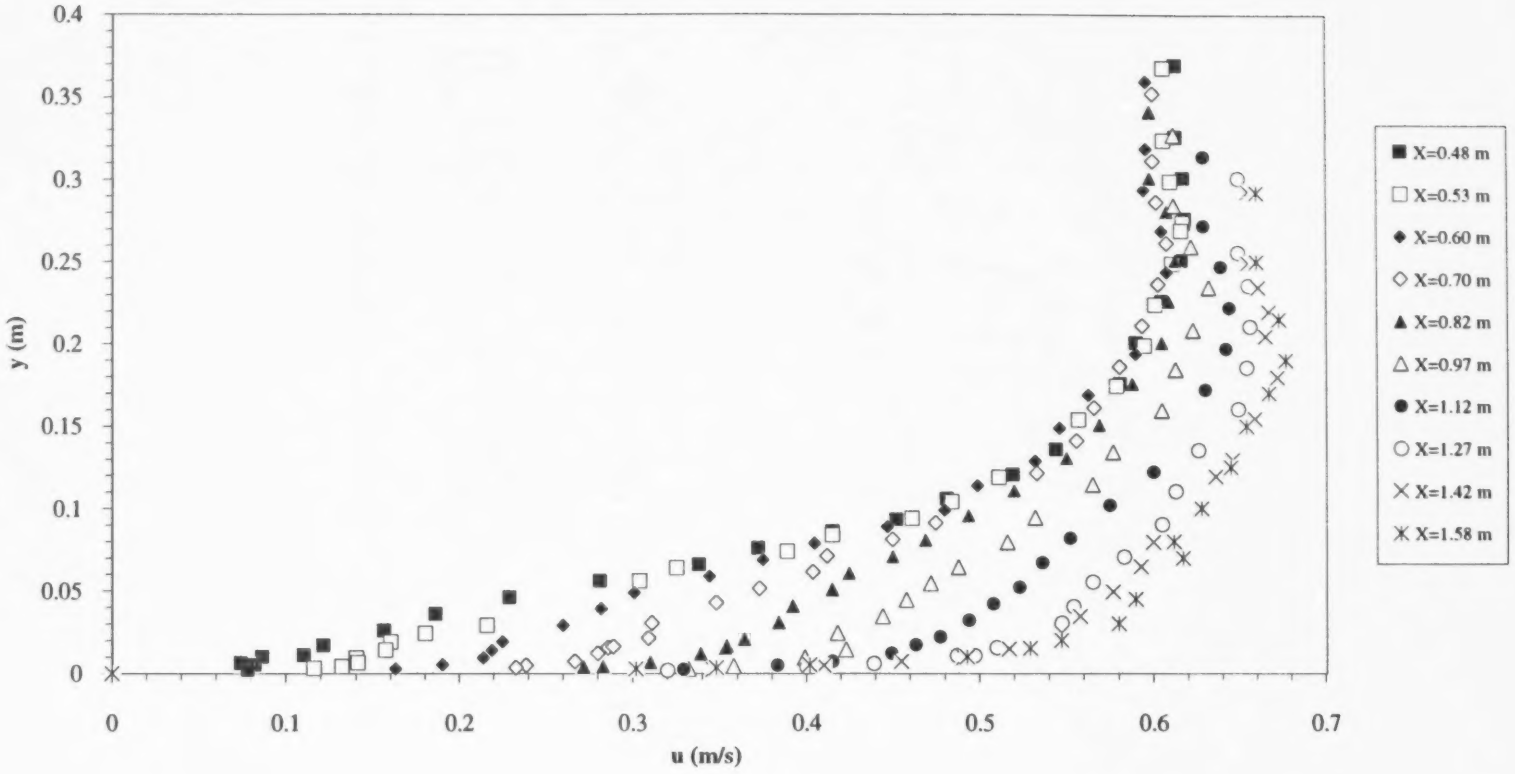


Figure 4.4 Velocity distribution in the wake region (T6)

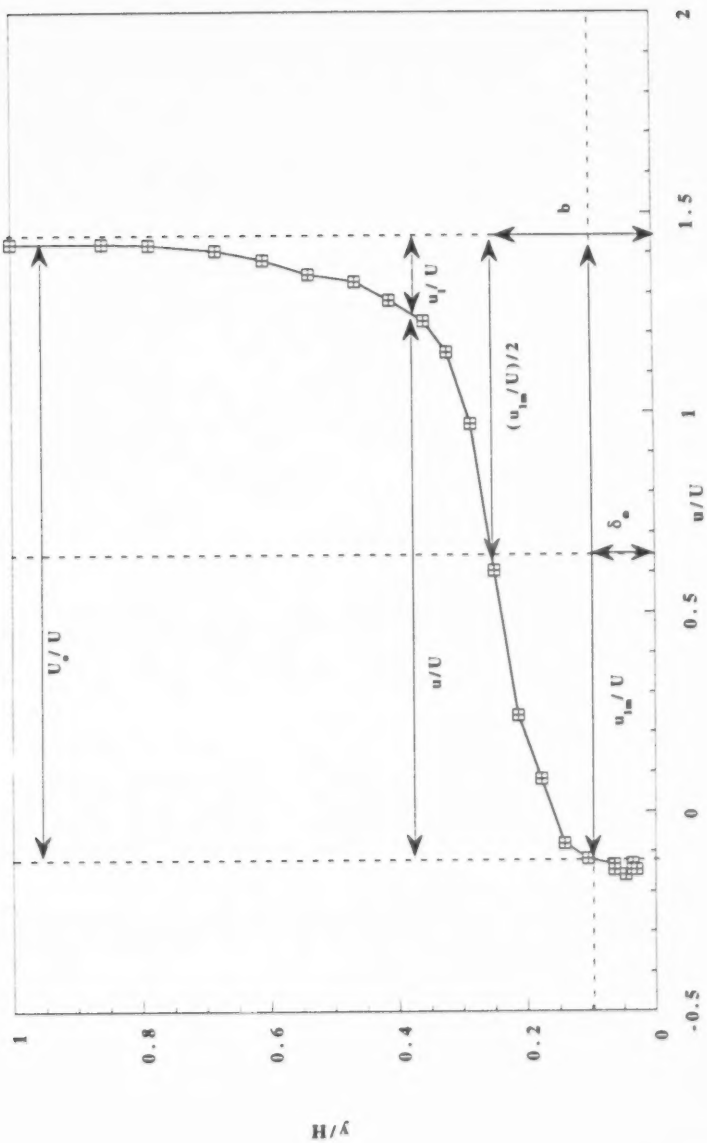


Figure 4.5 Determination of $u_{l\infty}$ and b

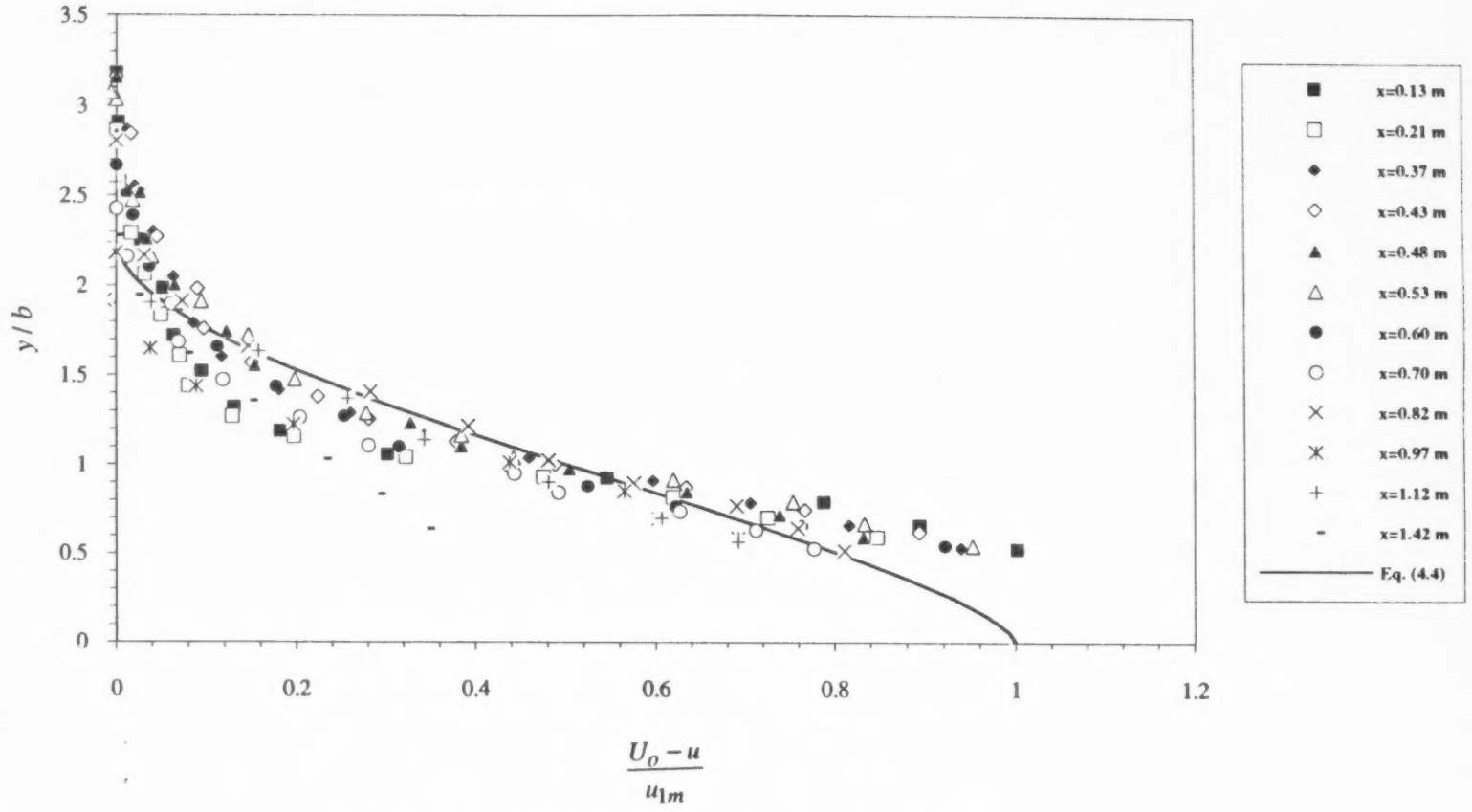


Figure 4.6 Velocity distribution : outer region (T5)

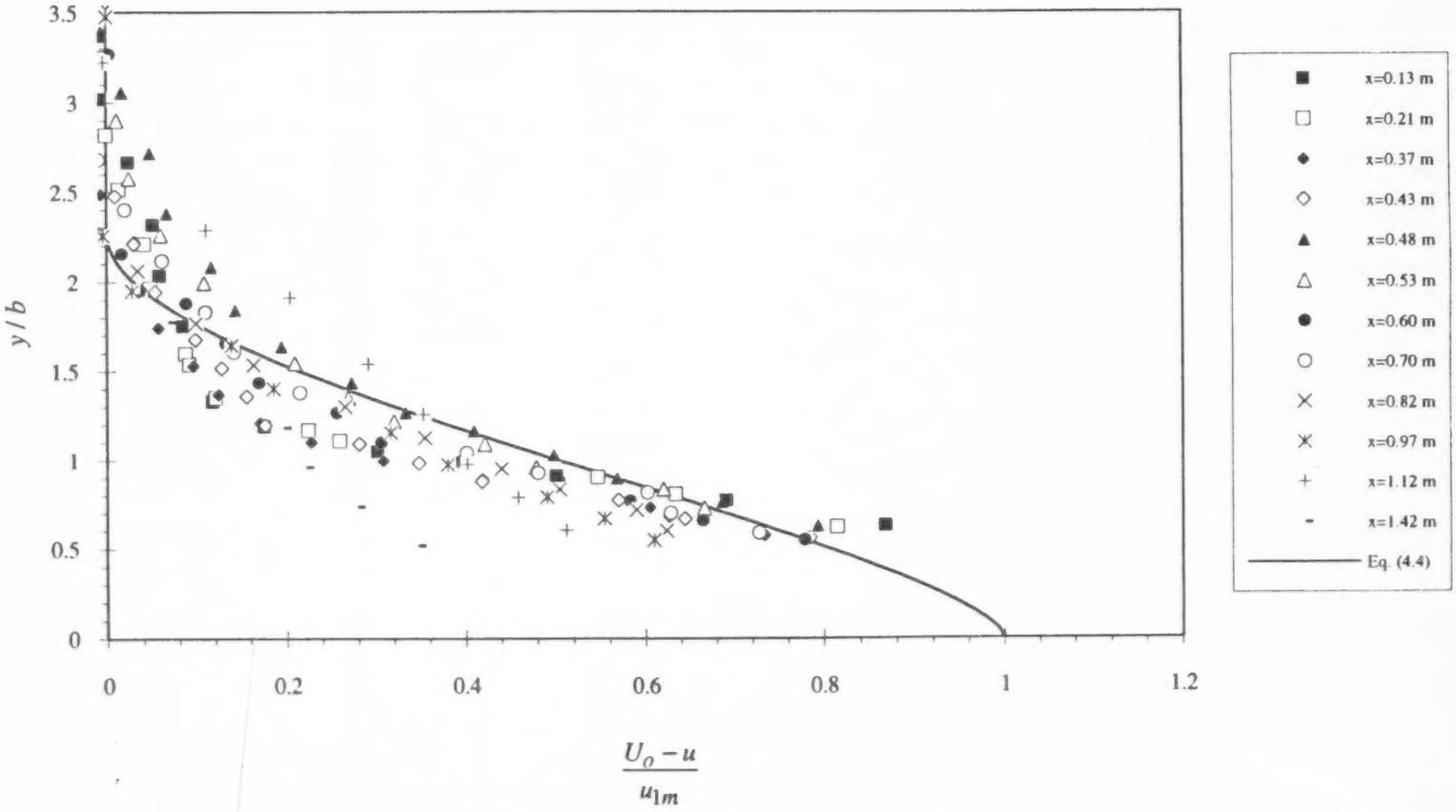


Figure 4.7 Velocity distribution : outer region (T6)

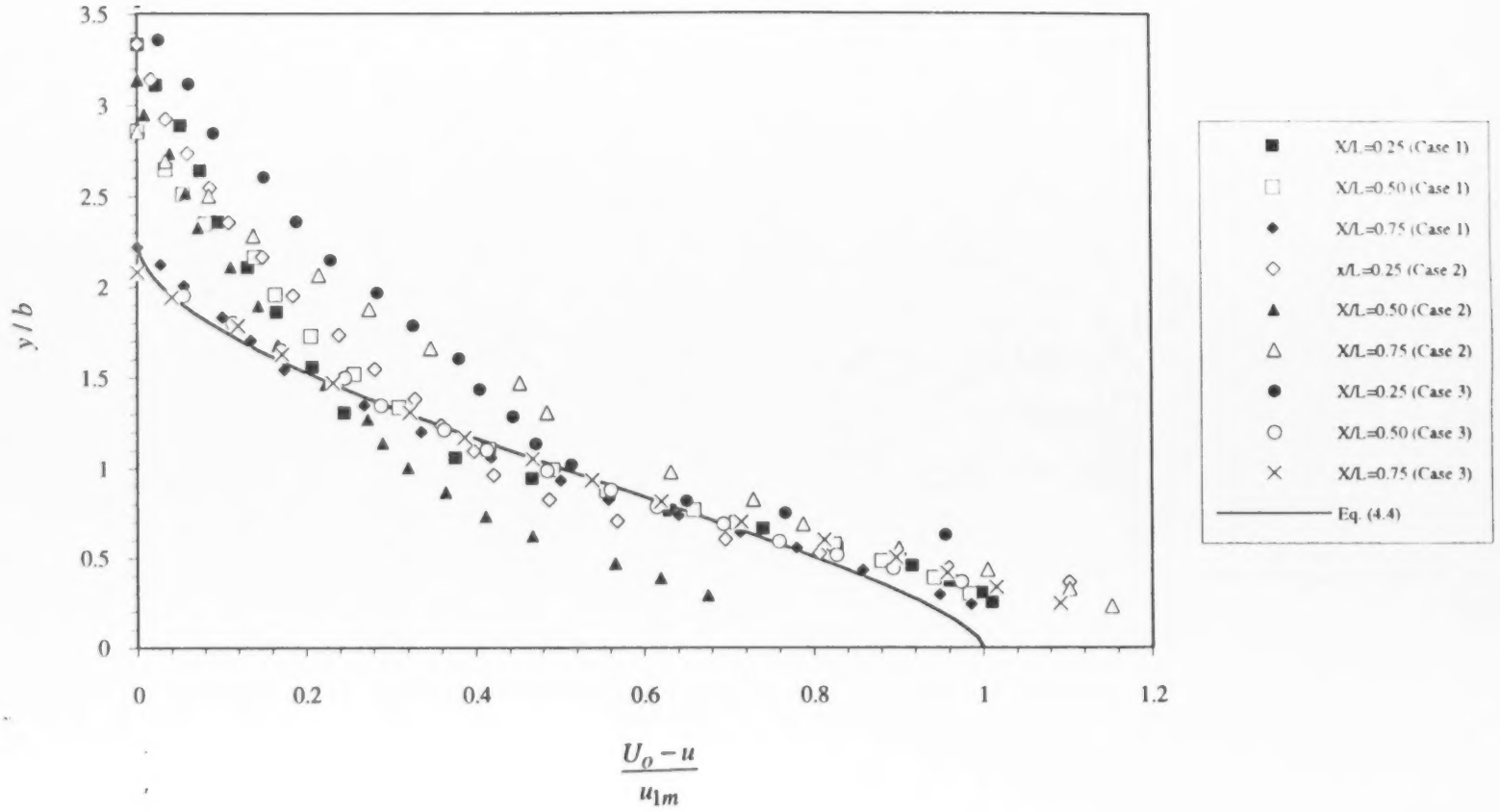


Figure 4.8 Velocity distribution : outer region (measured by D. Lyn)

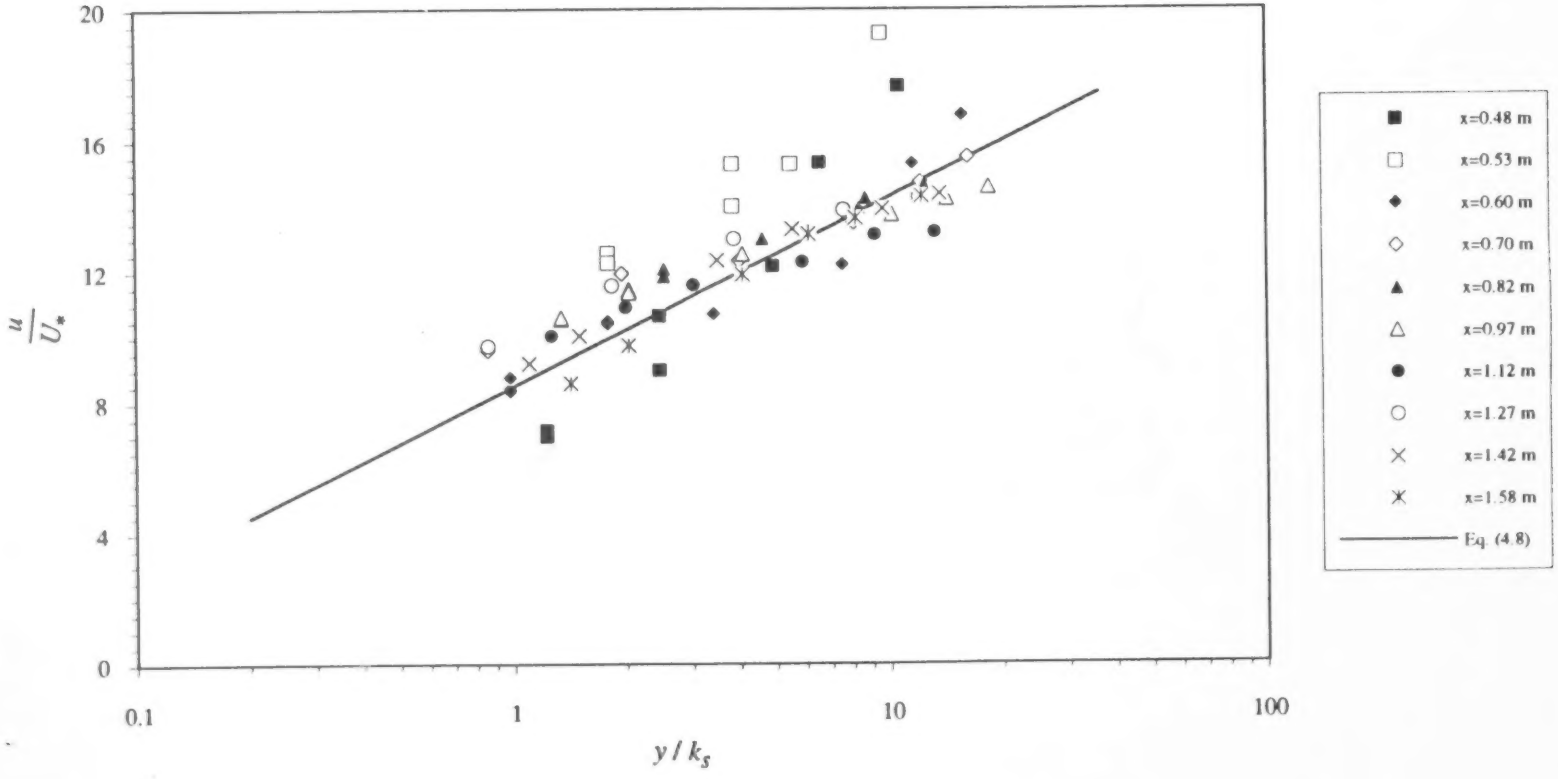


Figure 4.9 Velocity distribution : inner region (T5)

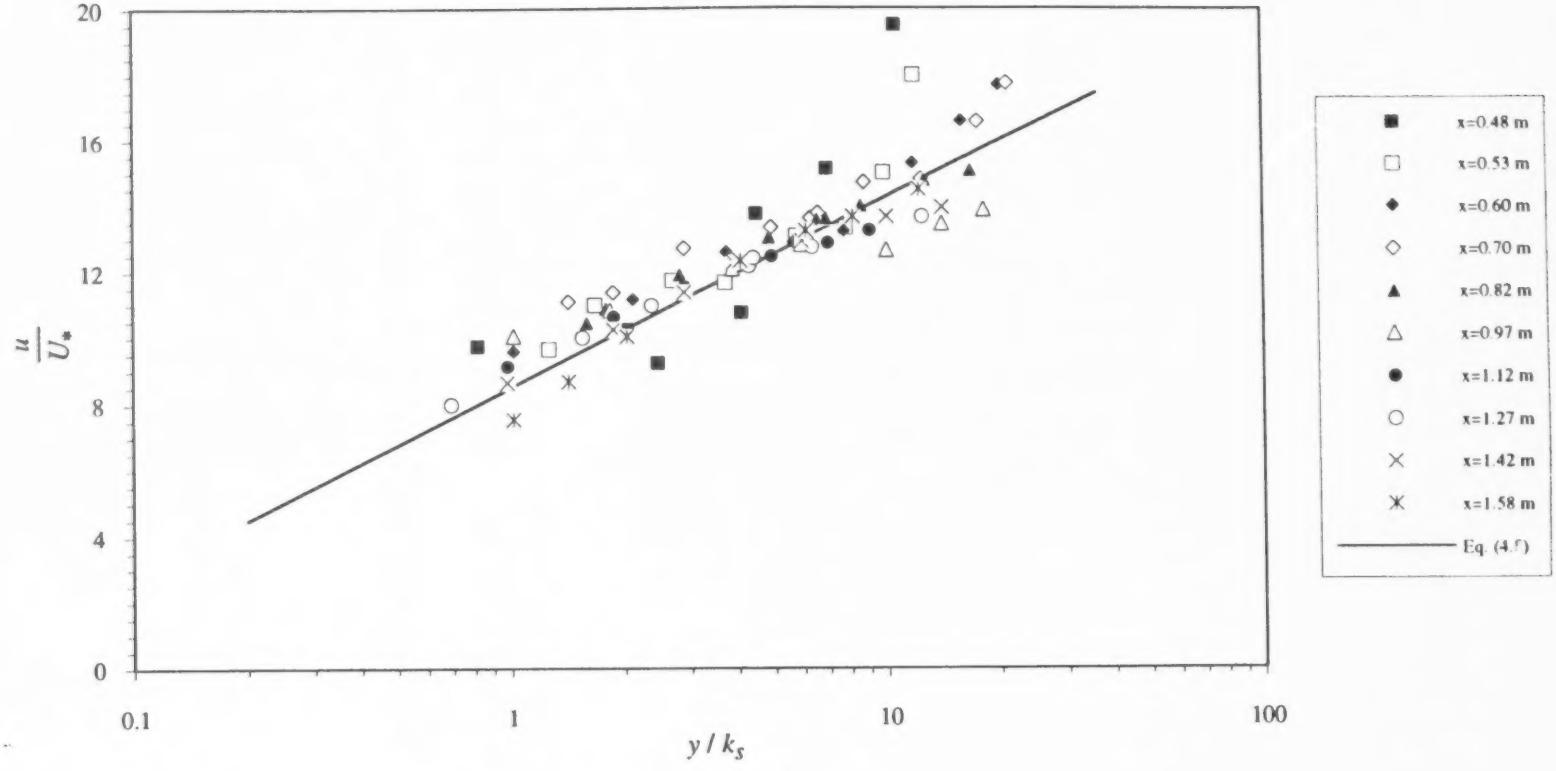


Figure 4.10 Velocity distribution : inner region (T6)

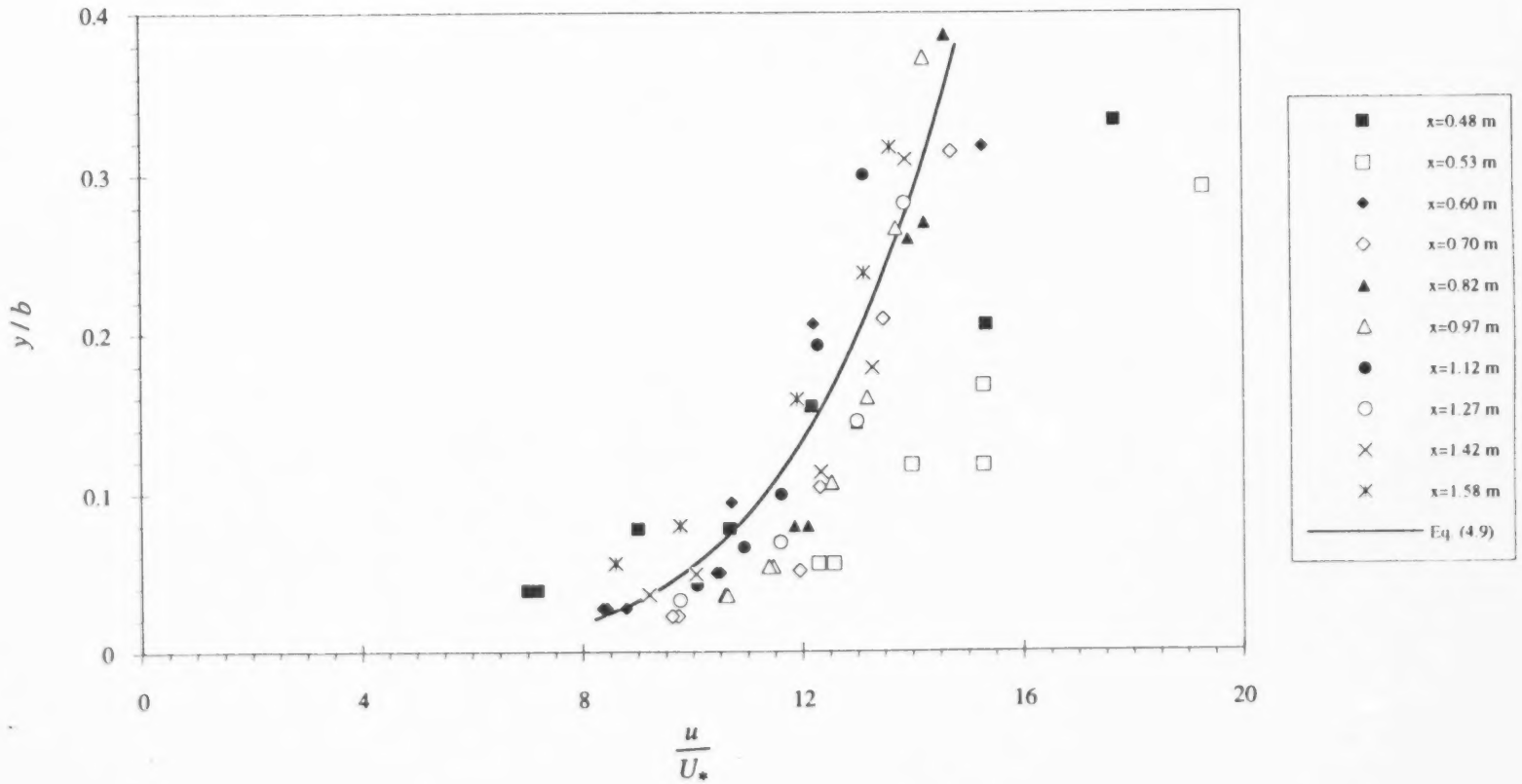


Figure 4.11 Power law form of velocity distribution : inner region (T5)

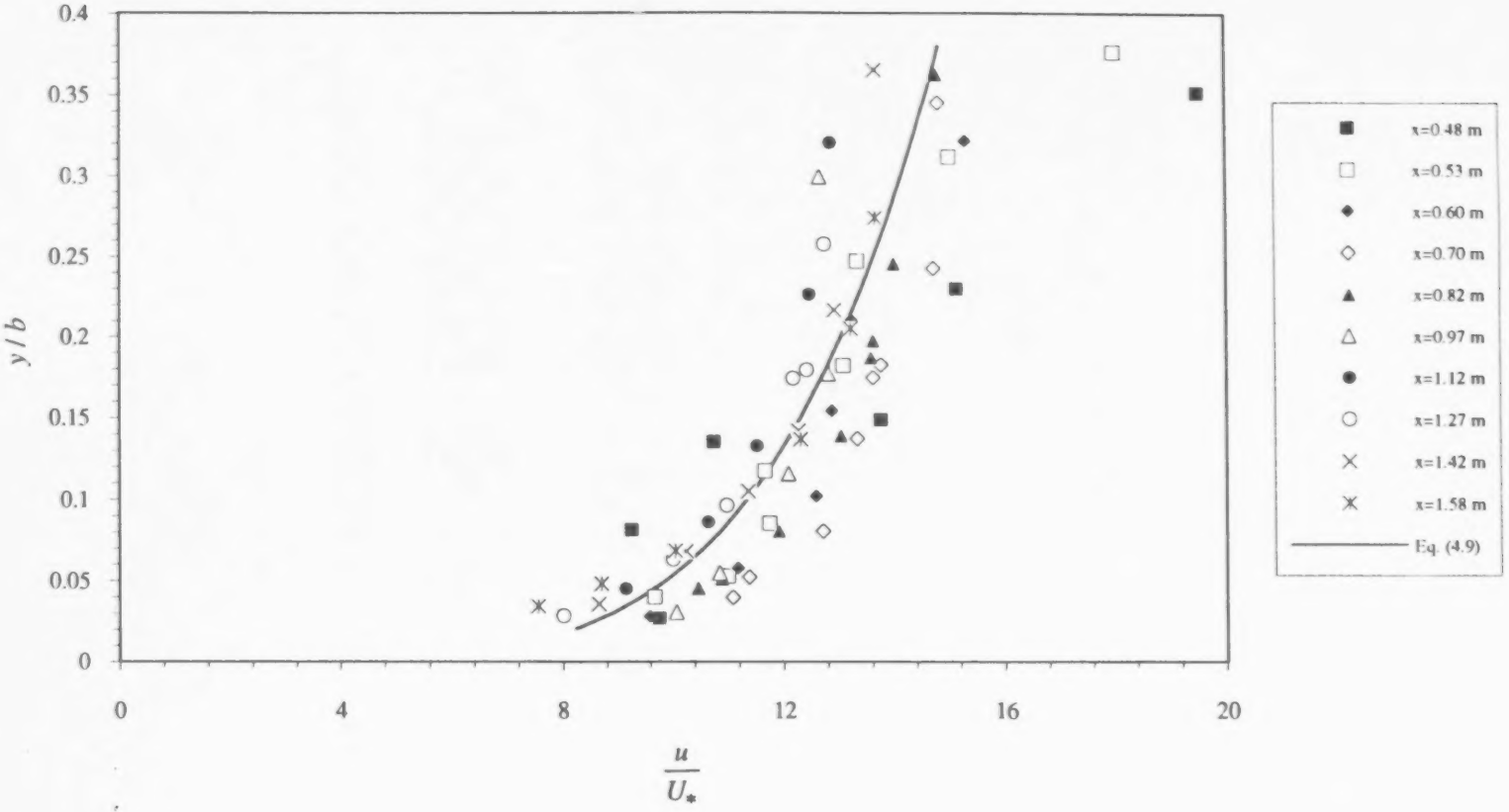


Figure 4.12 Power law form of velocity distribution : inner region (T6)

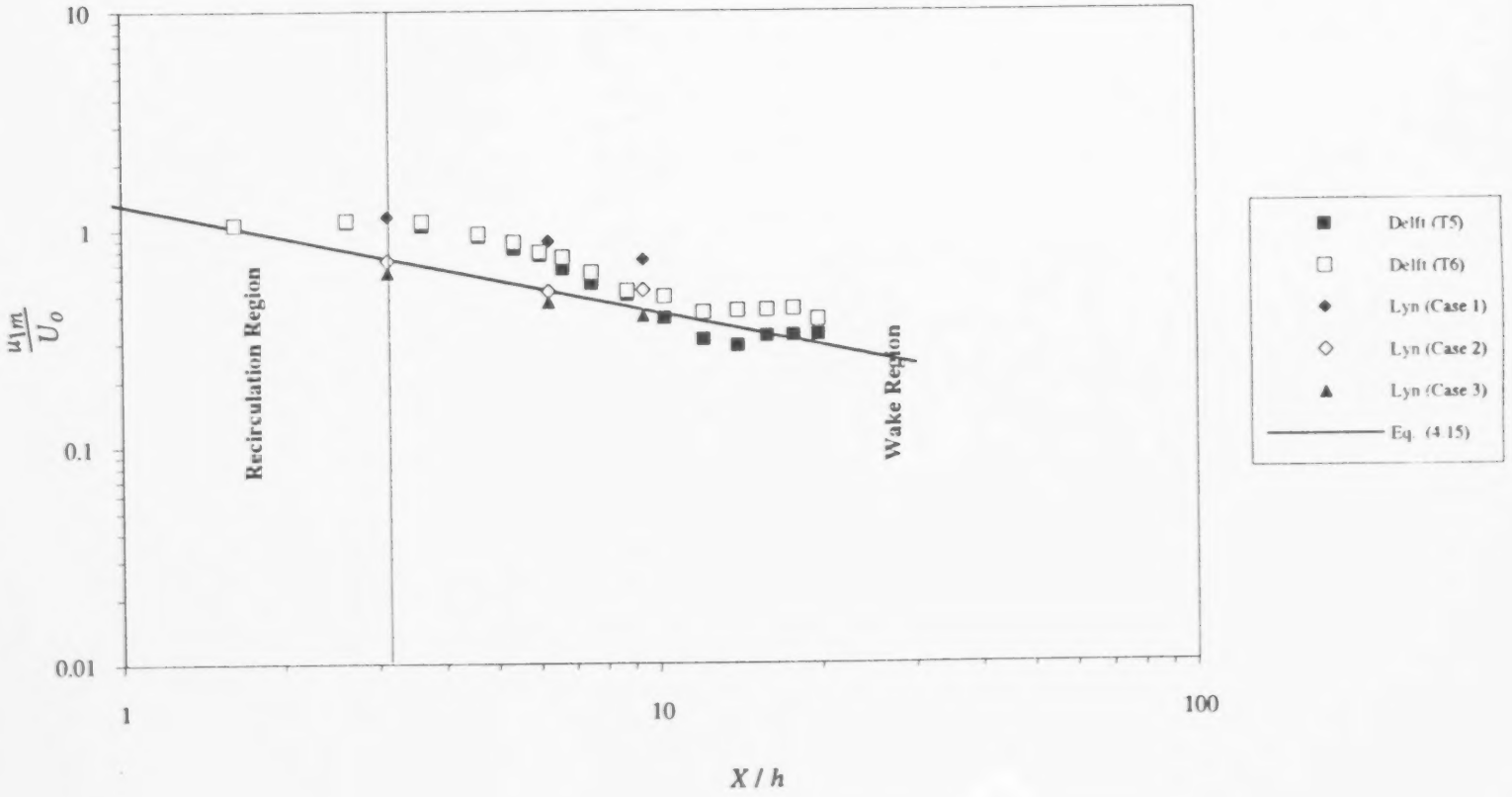


Figure 4.13 Decay of velocity scale

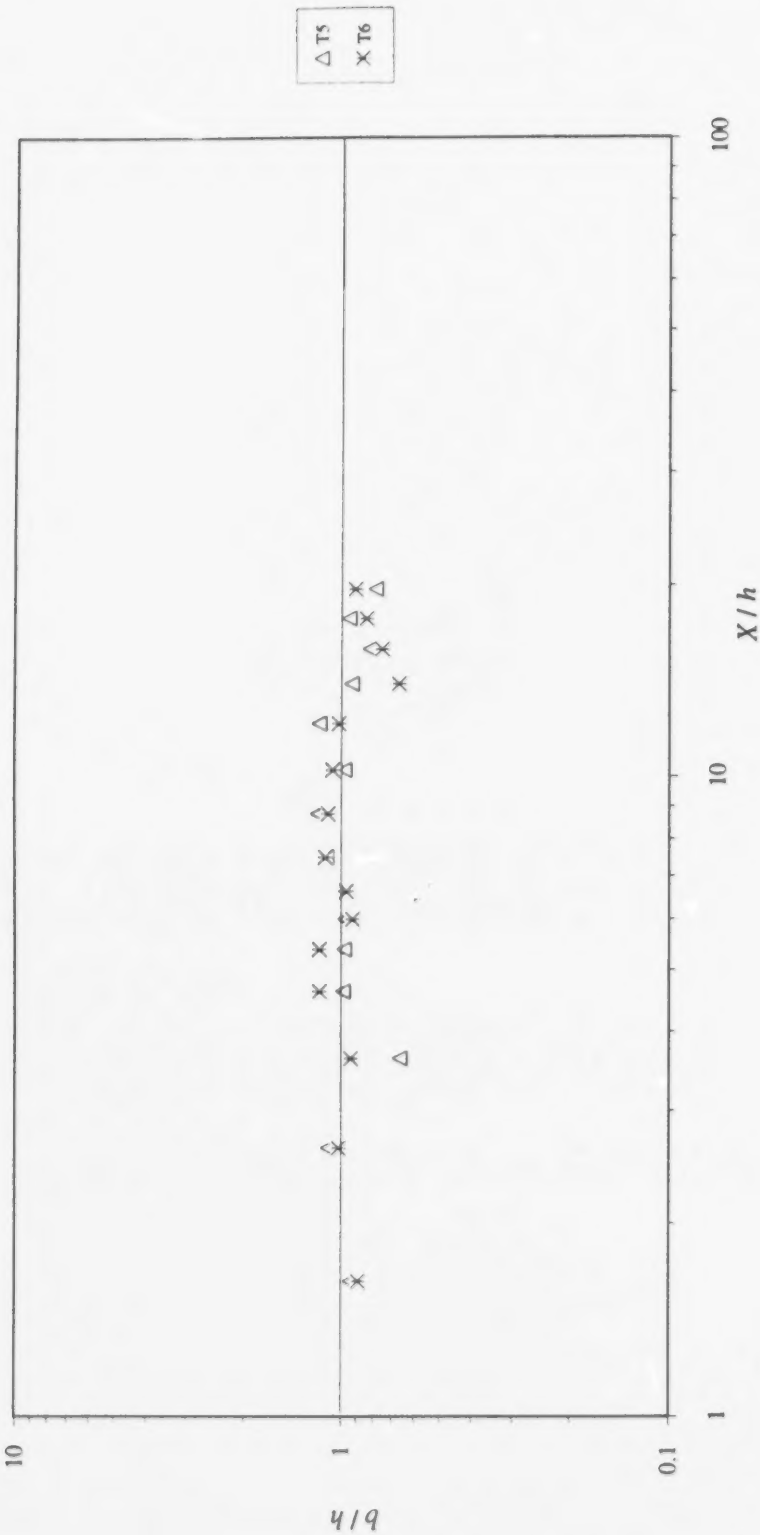


Figure 4.14 Growth of length scale

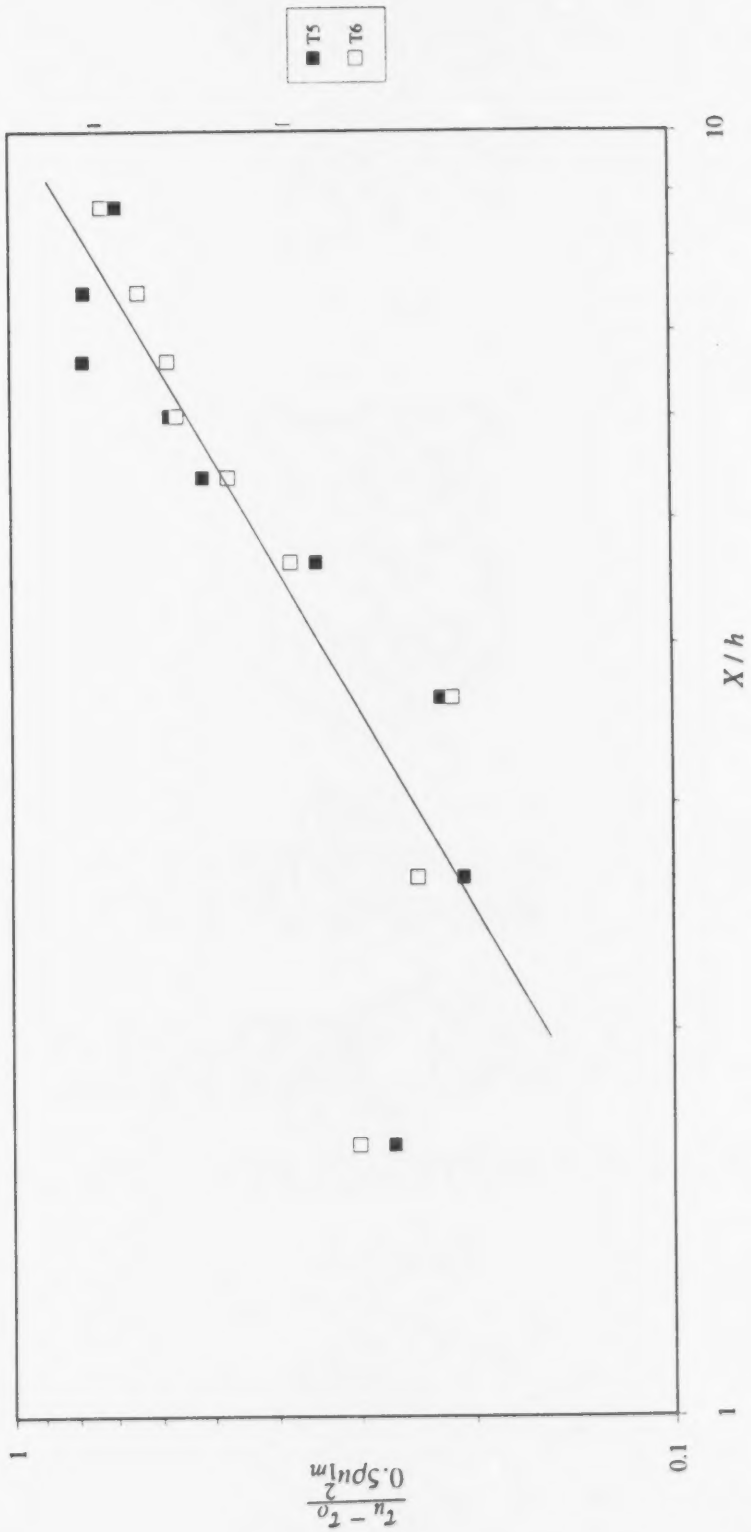


Figure 4.15 Variation of bed shear stress

CHAPTER 5: CONCLUSIONS

Velocity and bed shear stress distributions have been studied in fully developed turbulent flows over a fixed dune bed. The following results have been obtained from velocity profiles over dunes from non-dimensional analysis for velocity distribution and from the characteristics of the plane wall-wake.

-The value of the depth of the flow is one factor that affects the maximum defect velocity in turbulent flow and where the mean velocity occurs. As the depth of the flow increases, the maximum defect velocity close to the bed increases and the distance of the height $\left(\frac{y}{H}\right)$ of the mean velocity decreases.

-A review of the available experiments flow over dune indicates that the plane wall-wake is a good approach in describing the velocity distribution. The result of mean velocity profiles, wall shear stress, decay of the velocity defect, and growth of the half-width of the wake, are presented in convenient forms.

-The present results make it possible to make a fair estimate of the magnitude of longitudinal velocity for a particular channel at a certain location over dunes downflow from recirculation region.

-The bed shear stress distribution can not be reasonably evaluated from the wall-wake equation because the bed shear stress in the near-wake flow is changing rapidly due to the effect of vorticity, mixing, back flow in the recirculation region and the shear layer. As a result the far-wake approximation is not valid

-More experimental data on different h/H are required to find a model for longitudinal velocity distribution because approximations from the plane wall-wake are not valid over dunes with favorable pressure gradient.

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